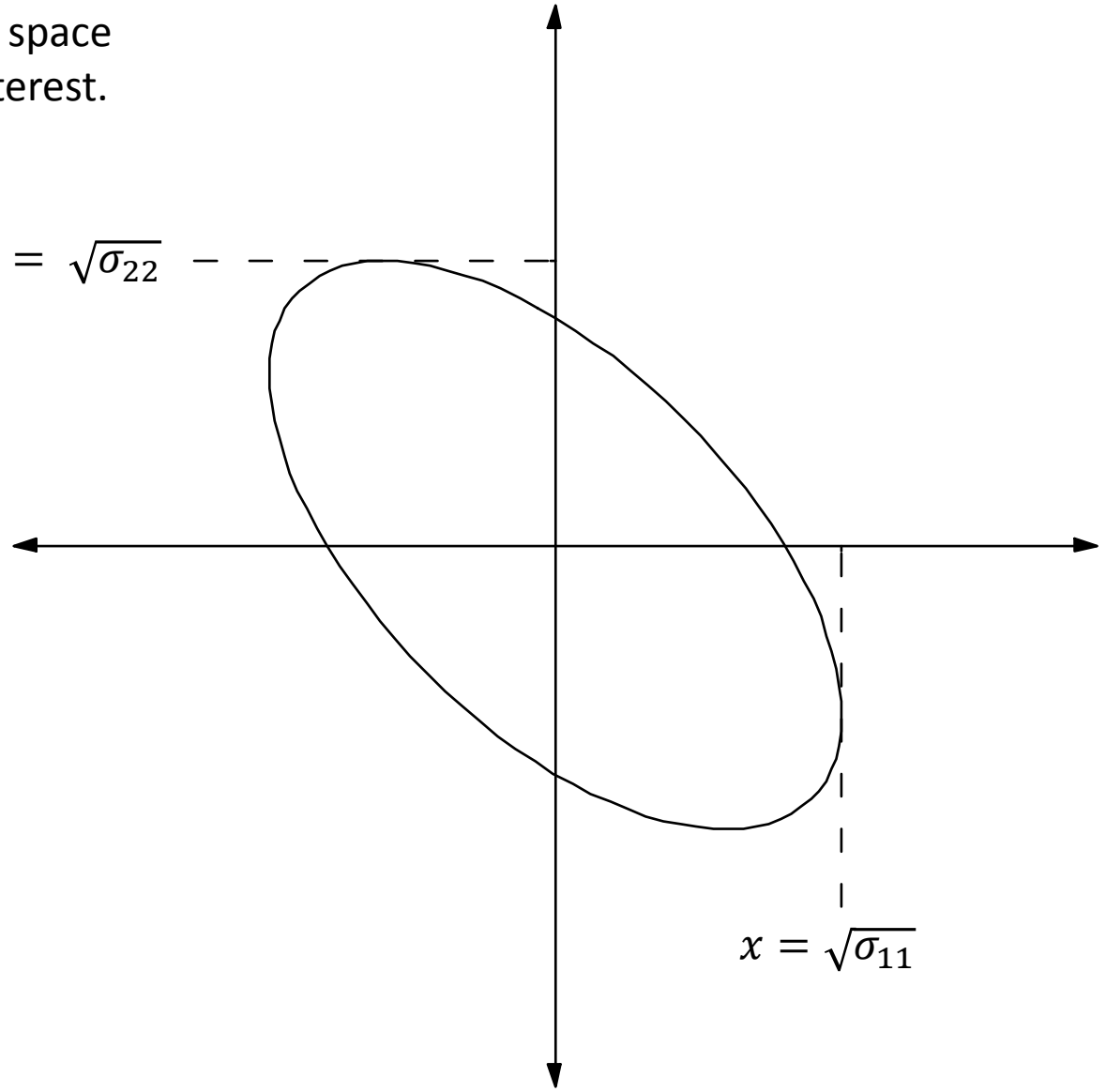
# Measuring the Beam Size

10/05/2015

Assume an ellipse in phase space contains the particles of interest.

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$



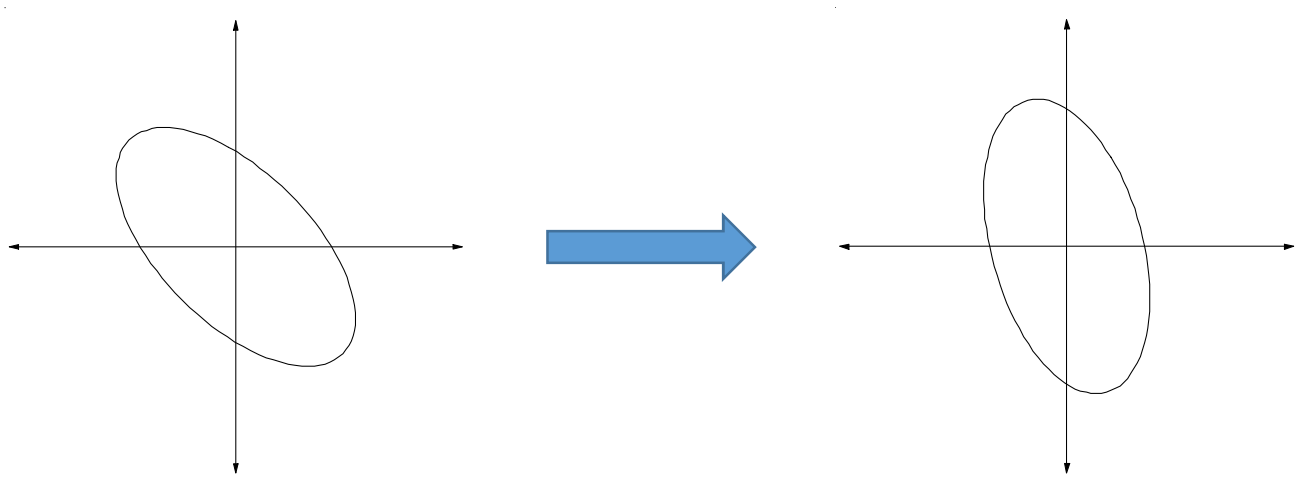
A drift (section of beamline with no magnetic elements) of length $l$ is represented by $L = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$

The new phase ellipse is calculated as follows:

$$\sigma_2 = L\sigma_1 L^T$$

Or

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}_2 = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}_1 \begin{pmatrix} 1 & 0 \\ l & 1 \end{pmatrix}$$



About the ellipse

$$\det(\sigma) \equiv \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix} = \sigma_{11}\sigma_{22}\text{-}\sigma_{12}\sigma_{21}$$

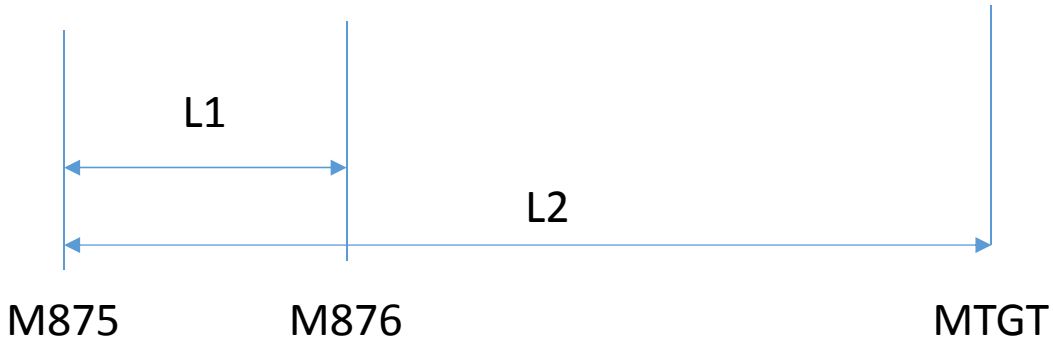
Area of the ellipse (emittance) $\varepsilon = \sqrt{\det(\sigma)} > 0$

$\sigma_{12} = \sigma_{21}$
$\sigma_{11} > 0$
$\sigma_{22} > 0$

For accelerator types:

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

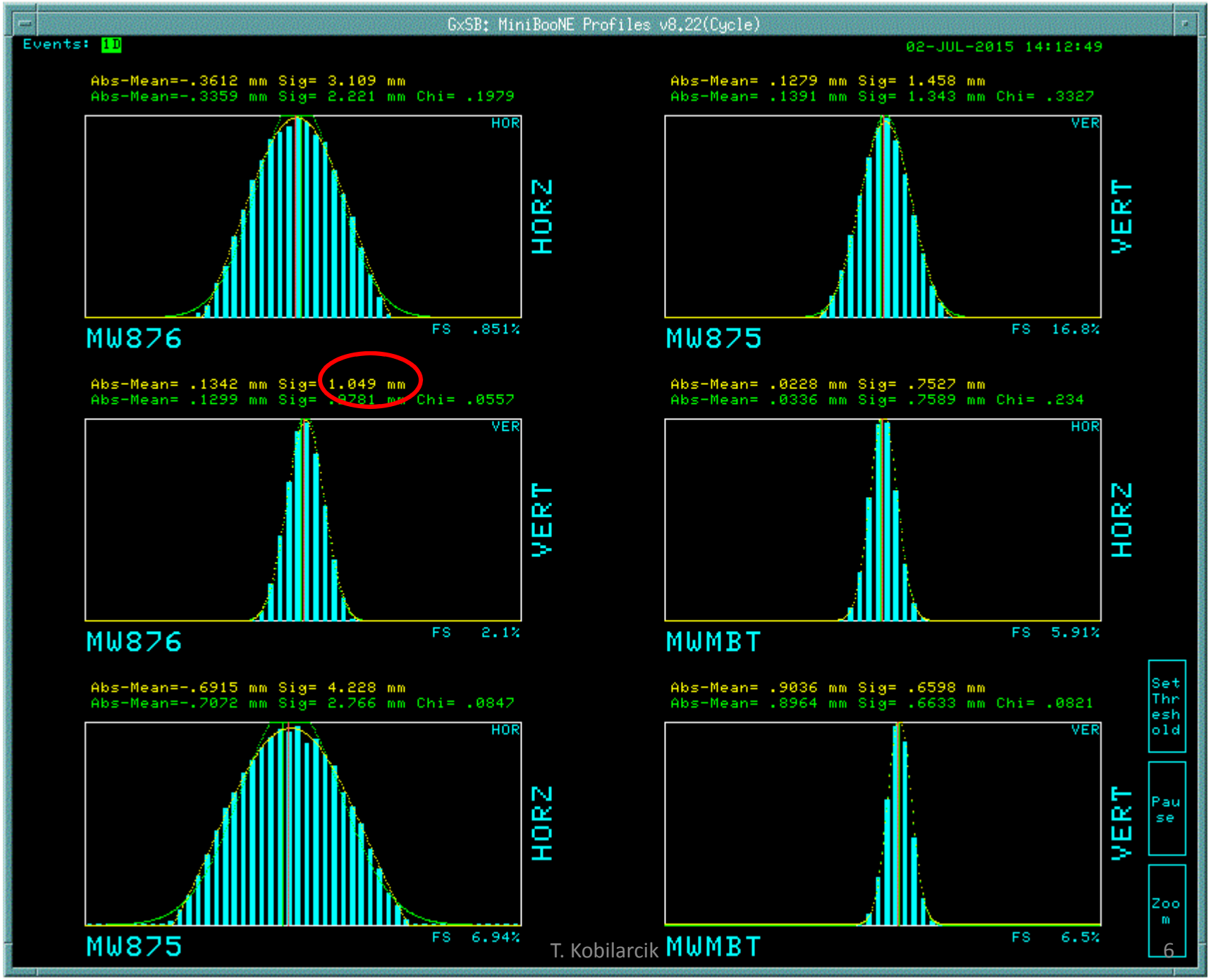
$$\begin{vmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{vmatrix} = 1$$




Writing it out long-hand:

$$\begin{pmatrix} \sigma_{11}(M875) \\ \sigma_{11}(M876) \\ \sigma_{11}(MTGT) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2l_1 & l_1^2 \\ 1 & 2l_2 & l_2^2 \end{pmatrix} \begin{pmatrix} \sigma_{11}(M875) \\ \sigma_{12}(M875) \\ \sigma_{22}(M875) \end{pmatrix}$$

So if one measures the beam width at three locations, one can then solve for the phase ellipse.


GxSB: MiniBooNE Profiles v8.22(Cycle)
Events: 10
02-JUL-2015 14:12:49
Abs-Mean=-.3612 mm Sig= 3.109 mm
Abs-Mean=-.3359 mm Sig= 2.221 mm Chi= .1979
HOR
HORZ
MW876
FS .851%
Abs-Mean= .1279 mm Sig= 1.458 mm
Abs-Mean= .1391 mm Sig= 1.343 mm Chi= .3327
VER
VERT
MW875
FS 16.8%
Abs-Mean= .1342 mm Sig= 1.049 mm
Abs-Mean= .1299 mm Sig= .9781 mm Chi= .0557
VER
VERT
MW876
FS 2.1%
Abs-Mean= .0228 mm Sig= .7527 mm
Abs-Mean= .0336 mm Sig= .7589 mm Chi= .234
HOR
HORZ
MWMBT
FS 5.91%
Abs-Mean=-.6915 mm Sig= 4.228 mm
Abs-Mean=-.7072 mm Sig= 2.766 mm Chi= .0847
HOR
HORZ
MW875
FS 6.94%
Abs-Mean= .9036 mm Sig= .6598 mm
Abs-Mean= .8964 mm Sig= .6633 mm Chi= .0821
VER
VERT
MWMBT
FS 6.5%
Set Threshold
Pause
Zoom

Three equations with three unkowns, and three measurements – so one can always solve it.
But does the answer make sense?

Horizontal: $13\pi$ mm*mr – pretty standard answer
Vertical: $4\pi$ mm*mr – kind of small