Drift and Diffusion of Electrons in Liquid Argon

What is known
Implications for mBooNE TPC
What we might want to know

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μBooNE Collaboration Meeting

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See also doc# 280-v2 by Kirk McDonald

Drift and Diffusion of Electrons in Liquid Argon

What is known, i.e. measured

Drift Literature:

1. W. Walkowiak, *Drift velocity of free electrons in liquid argon*, NIM **A449** (2000) 288.
2. S. Amoruso, *et al.*, *Analysis of the liquid argon purity in the ICARUS T600 TPC*, NIM **A516** (2004) 68.
3. S.S-S.Huang and G.R. Freeman, *Electron transport in gaseous and liquid argon: effect of density and temperature*, Phys. Rev. **A24** (1981) 714.
4. H. Schnyders, *et al.*, *Electron drift velocities in liquified argon and krypton at low electric field strengths*, Phys. Rev. **150** (1966) 127.

Plus at least 14 others…

Diffusion Literature:

1. E. Shibamura, *et al.*, *Ratio of diffusion coefficient to mobility for electrons in liquid argon*, Phys. Rev. **A20** (1979) 2547. (many fields > 1.5 kV/cm)
2. S.E. Derenzo, LBL Physics Note No. 786 (1974) unpublished . *(at 1.4 & 2.7 kV/cm, large error bars?)*
3. S.E. Derenzo *et al.*, *Test of a liquid argon ion chamber with a 20mm RMS resolution*, **NIM** 122 (1974) 319 . *(at 2.7kV/cm only)*

That’s it for diffusion!

The Relationship Between Drift and Diffusion

The Einstein–Smoluchowski Ratio

The Foundation:

1. A. Einstein, *Über die von der molekularkinetischen Theorie geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen*, Annalen der Physik **322**(1905) 549. *(see p. 555 bottom)*

2. M. von Smoluchowski, *Zur kinetischen Theorie der Brownschen Molekularbewegung und der Suspensionen*, Annalen der Physik **21** (1906) 756.

They found that for particles (x) in thermal equilibrium with a medium (M), the diffusion constant, mobility, and temperature were related:

$$kT_M = \frac{eD_x}{\mu_x}$$

If thermal equilibrium is not obtained (as in drift in high E fields), we can use the E-S ratio to define an electron energy:

$$\varepsilon_e \equiv \frac{eD_e}{\mu_e}$$

Note that the mean electron energy differs from this energy:

$$\varepsilon_M \equiv \int \varepsilon \rho_{EEDF}(\varepsilon) d\varepsilon = \tfrac{3}{2}kT$$

What is Known about Diffusion of Electrons in Liquid Argon

Two measurements

From Shibamura paper, showing also the data of Derenzo (unpublished)

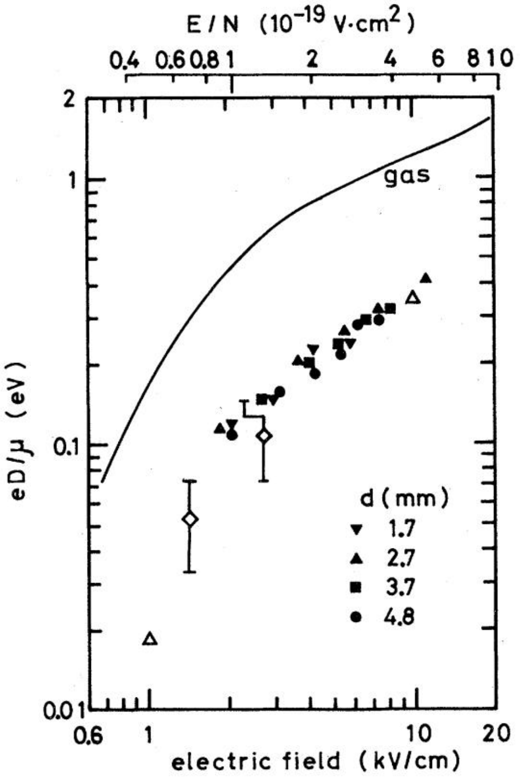


FIG. 7. Field dependence of the ratio eD/μ in liquid argon. The symbols ▲, ▼, ■, and ● represent the present results. Points ◇ and △ are the results according to Derenzo (Ref. 9) and Lekner (Ref. 7), respectively. Solid curve shows the results for gaseous argon. In the upper horizontal scale, N is the atomic density in liquid argon (2.1×10^{22} cm^{-3}).

“Leckner” points (triangles) are plotted at 2/3 of the calculated mean electron energy obtained from a solution to the Boltzmann transport equation in liquid Ar.

See Table I in Lekner, PR. **158** (1967) 130.

At thermal equilibrium kT=0.0075 eV

Diffusion of Electrons in Strong Electric Fields Is Not Isotropic

Theory – Boltzmann Transport Equation

1. H.R. Skullerud, *Longitudinal diffusion of electrons in electrostatic fields in gases*, J. Phys. B**2** (1969) 696.
2. J.H. Parker and J.J. Lowke, *Theory of electron diffusion parallel to electric fields. I. Theory*, Phys. Rev. **181** (1969) 290.

The essence, from Parker and Lowke:

If the electron collision frequency increases with energy, then the leading edge of the electron swarm has a reduced mobility because it has a higher-than-average electron energy and therefore higher collision rate. Conversely for the trailing edge, and the swarm is “compacted” along the drift direction.

From Skullerud:

collision rate proportional to a power of velocity:

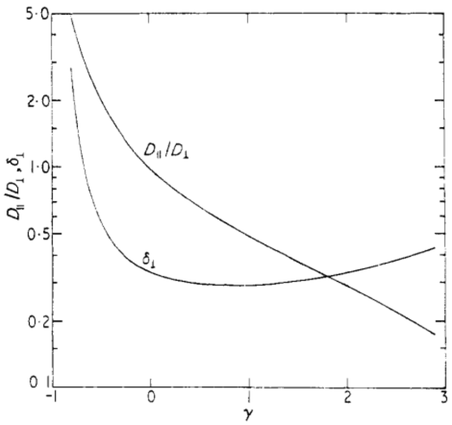


Figure 2. The dimensionless diffusion coefficient $\delta_{\perp}$ (equation (59)) and the ratio $D_{\parallel}/D_{\perp}$ of the longitudinal to the lateral diffusion coefficient for the case of a strong electric field and a collision frequency $\nu_M \propto v^{\gamma}$, as functions of γ.

mean free path independent of velocity:

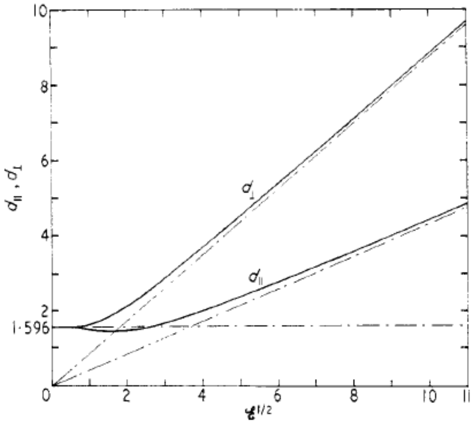


Figure 1. The dimensionless diffusion coefficients $d_{\parallel} = ND_{\parallel}\sigma_M(kT/m)^{-1/2}$ and $d_{\perp} = ND_{\perp}\sigma_M(kT/m)^{-1/2}$ as functions of the dimensionless field parameter $\mathscr{E} = (E/N)(e/kT\sigma_M)(M/m)^{1/2}$. Asymptotic values for large and small $\mathscr{E}$ are shown as broken lines.

Diffusion of Electrons in *Gases*

Measurements and Theory show $D_L/D_T < 1$

Selected Literature:

1. J.J. Lowke and J.H. Parker, *Theory of electron diffusion parallel to electric fields. II. Application to real gases*, Phys Rev **181** (1969) 290.
2. The code Magboltz, S.F. Biagi, *Monte Carlo simulation of electron drift and diffusion in counting gases under the influence of electric and magnetic fields*, NIM **A421** (1999) 234.

Lowke and Parker

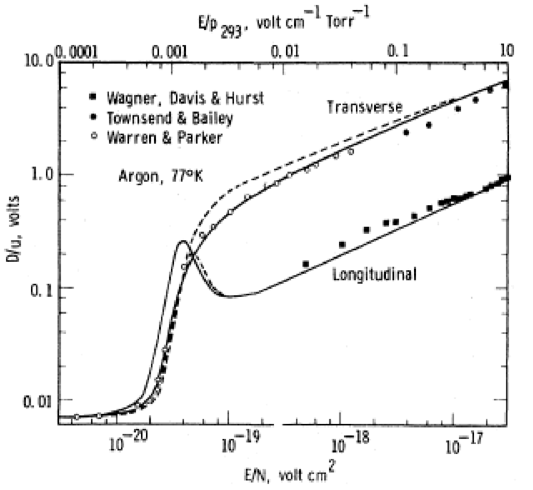
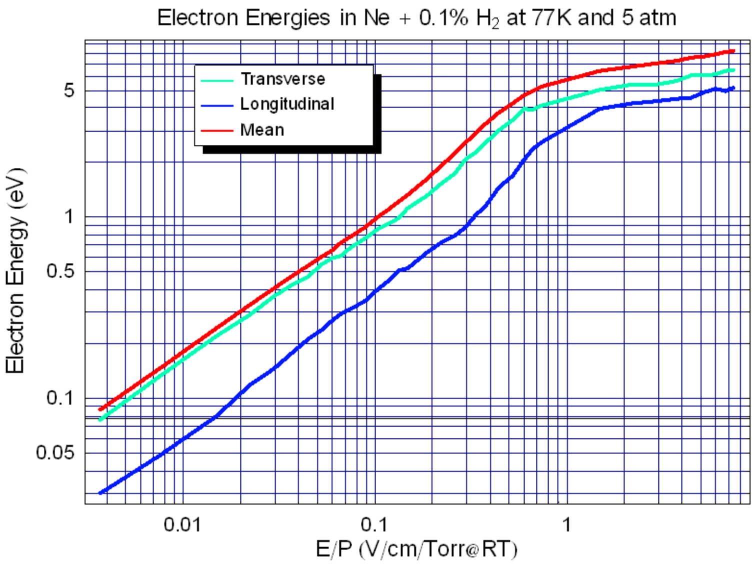


FIG. 3. Theoretical and experimental values of D_T/μ and D_L/μ for argon at 77°K. The full curve is obtained using as input the momentum transfer cross section of Engelhardt and Phelps, and the broken curve from using the momentum transfer cross section of Golden.

Calculation from Magboltz7



Theory of Transport of Electrons in Non-polar Liquids

What is known

Highly Selected Literature:

1. M.H. Cohen and J. Leckner, *Theory of hot electrons in gases, liquids, and solids*, PR **158** (1967) 305.
2. J. Lekner, *Motion of electrons in liquid Argon*, Phys. Rev. **158** (1967) 130.
3. B.M. Smirnov, *Kinetics of electrons in gases and condensed systems*, Phys.-Uspekhi **45** (2002) 1251.
4. V.M. Atrazhev and I.V. Timoshkin, *Transport of electrons in atomic liquids in high electric fields*, IEEE Trans. on Dielectrics **5** (1968) 450.

In liquids the theory is more complex than in gases because the scattering of electrons occurs from correlations of atoms and from density fluctuations – one result is that the cross section and therefore the mobility is higher in liquid.

$$\sigma_{eff}(\varepsilon, N) = \sigma_{cell}(\varepsilon, N) + \sigma_{f}(\varepsilon, N)$$

Theory of Atrazhev and Timoshkin

Calculations and measurements of mobility and electron temperatures for liquid argon near the triple point and near the critical point

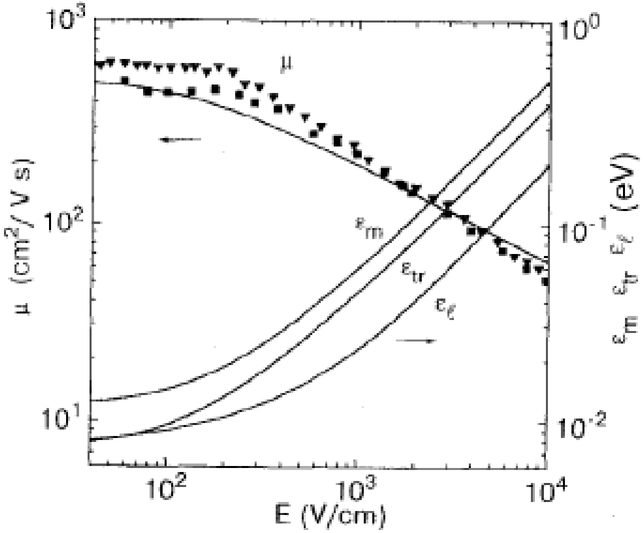


Figure 4. Transport coefficients of electrons in liquid Ar near the triple point ($N_{tp} = 2.2\times10^{22}$ cm^{-3}) as a function of electric field strength. Points, experimental data for mobility [6, 8]; solid lines, the calculated mobility μ, the mean electron energy ϵ_m, and the transverse ϵ_{tr} and the longitudinal ϵ_ℓ characteristic energies.

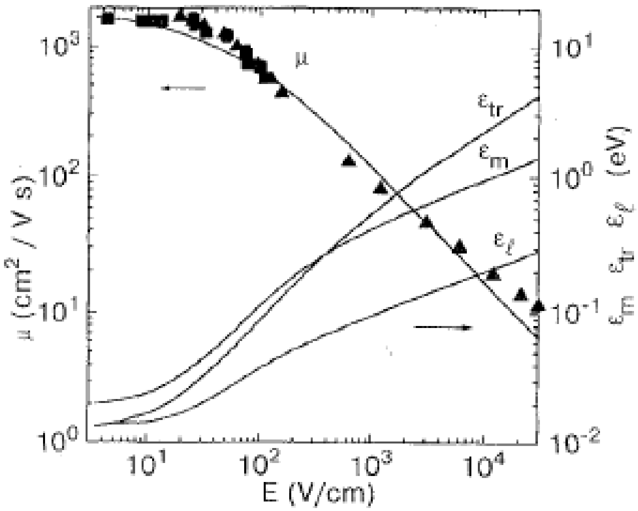


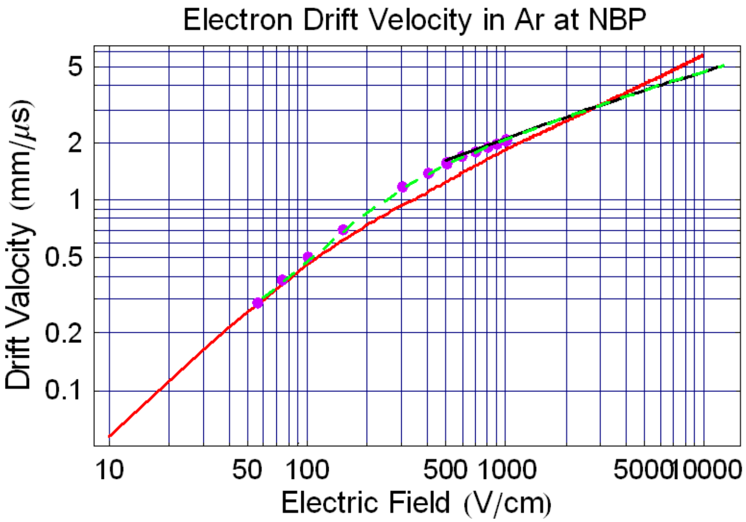
Figure 5. Transport coefficients of electrons in liquid Ar at the density $N_m = 1.2\times10^{22}$ cm^{-3} as a function of electric field strength. Points, experimental data for mobility [1, 6, 9]; solid lines, the calculated mobility, the mean electron energy ϵ_m, and the transverse ϵ_{tr} and the longitudinal ϵ_l characteristic energies.

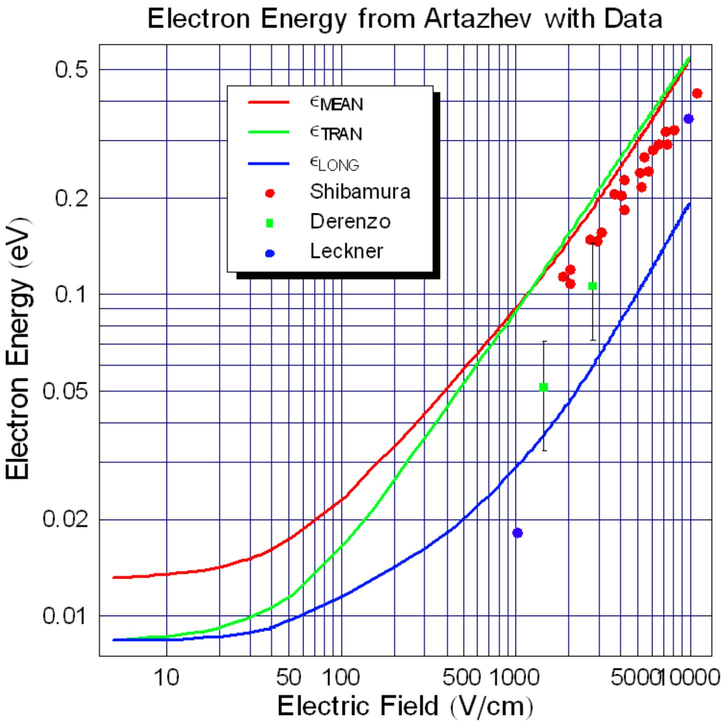
Mobility data is from Huang and Freeman and from H. Schnyders, *et al*.

Drift Velocity and Electron Energies in LAr

Interpolated to normal boiling point from A&T theory

Solid lines are Atrazhev calculations





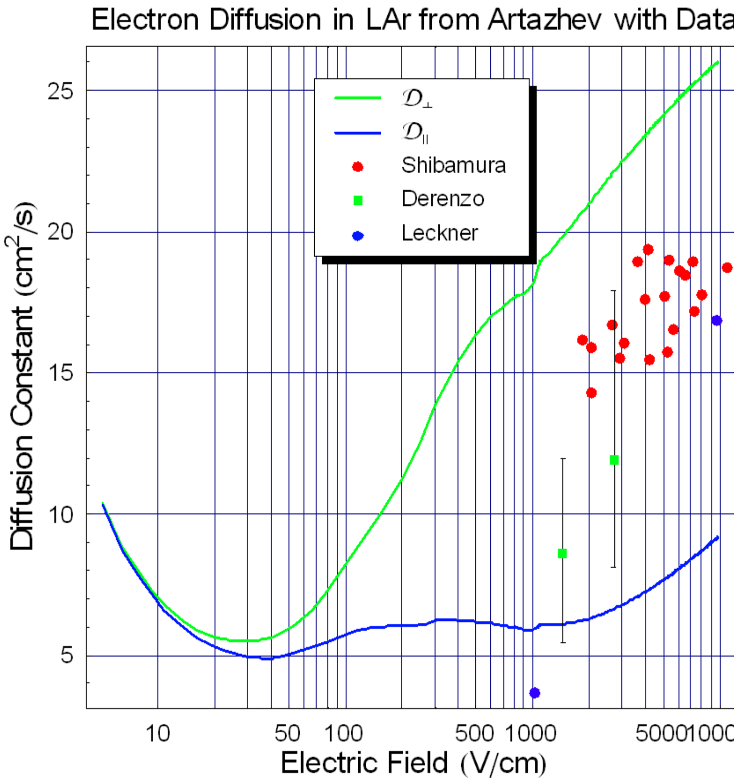
Mobility data is from Walkowiak and from Amoruso, *et al*. (dashed line is fit to both)

Transverse diffusion data is from Shibamura, *et al*.

Note that as $E \to 0 \Rightarrow \varepsilon_T = \varepsilon_L = \frac{2}{3}\varepsilon_M$

Transverse and Longitudinal Diffusion in LAr

Interpolated to normal boiling point from A&T theory



At 500 V/cm we find: D_T=16.3 cm^2/s, D_L=6.2 cm^2/s from theory; for TE D=2.45 cm^2/s

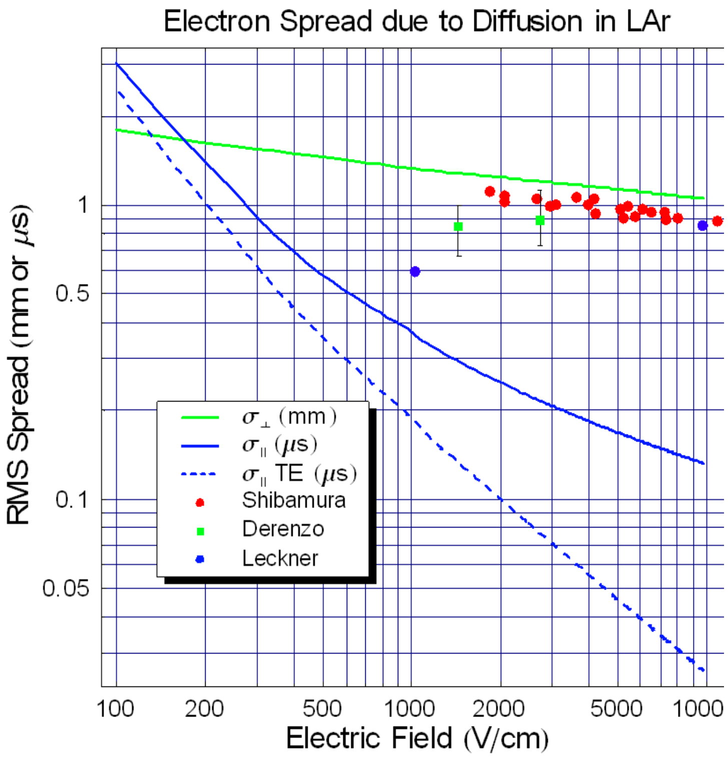
But note that for D_T, theory is above measurements. Perhaps this is true for D_L too!

Note also that ICARUS states that $D_{(?)}$=4.8 cm^2/s, without attribution; See

S. Amerio, et al., *Design, construction, and tests of the ICARUS T600 detector*, NIM **A527** (2004) 329.

Drift and Diffusion of Electrons in Liquid Argon

Implications for μBooNE TPC



$$\sigma = \sqrt{\frac{2Dz}{v}} = \sqrt{\frac{2\varepsilon z}{E}}$$

$v_D = 1.54$ mm/μs, $\mathcal{D}_\perp = 16.3$ cm^2/s, and $\mathcal{D}_\parallel = 6.2$

RMS electron swarm sizes after drift for a field of

Distance (m)	$\sigma_\perp$ (mm)	$\sigma_\parallel$ (μs)	$\sigma_\parallel$T
0.25	0.73	0.29	0.1
0.50	1.03	0.41	0.2
0.75	1.26	0.50	0.3
1.00	1.46	0.58	0.3
1.25	1.63	0.65	0.4
1.50	1.78	0.71	0.4
1.75	1.93	0.77	0.4
2.00	2.06	0.82	0.5
2.25	2.18	0.87	0.5
2.50	2.30	0.92	0.5

1. Probably do not want to shape for more than 0.4 μs.
2. Only a small benefit is possible from decreasing the wire plane spacing.
3. Such a decision should be guided by a detailed analysis of the estimated signal sizes and shapes, including all noise sources.

Drift and Diffusion of Electrons in Liquid Argon

What we might want to know

We should actually *measure* the longitudinal diffusion in a liquid, and also *measure* the transverse diffusion in liquid argon *at our fields* (Shibamura stopped at ~1.5 kV/cm).

As a member of the eBubble Collaboration, I am also interested in drift and diffusion of electrons, both free and localized in bubbles, in supercritical neon, so …

Francesco and I propose to use the eBubble pressure vessel and cryostat to measure drift velocity and transverse and longitudinal diffusion of electrons in neon and argon over a wide range of electric fields.

For the more immediate problem of understanding whether the wire plane geometry can be further optimized, I propose to work with Bo to model the full signal formation chain, including noise sources, with the goal of predicting the “track spread function”.