Path Length of Muons Traversing an Arbitrary Volume

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1 Problem

Deduce the total path length of muons traversing a volume V of arbitrary shape in terms of the rate $dR/d\cos\theta$, whose units are number/cm^2/s, of muons incident on a horizontal surface at the location of the volume under consideration, where θ is the polar angle of the muon trajectory with respect to the vertical. You may ignore the energy loss by the muons in the volume and suppose that all muons entering the volume exit it after following a straight-line trajectory within.

2 Solution

The difficulty in accounting for muons that enter or leave the sides of the volume can be avoided by subdividing it into a set of horizontal lamina of vertical thickness t that is small compared to the characteristic length of the volume. Then, to a good approximation every muon that enters a lamina from the top exits it from the bottom. If the muon trajectory makes angle θ to the vertical, then its path length within the lamina is $t/\cos\theta$.

The total path length of muons traversing a lamina of area A in one second is

$$\int_0^1 \frac{At}{\cos\theta}\frac{dR}{d\cos\theta}\, d\cos\theta = RAt\left\langle\frac{1}{\cos\theta}\right\rangle, \tag{1}$$

where

$$\left\langle\frac{1}{\cos\theta}\right\rangle = \frac{1}{R}\int_0^1 \frac{1}{\cos\theta}\frac{dR}{d\cos\theta}\, d\cos\theta. \tag{2}$$

Summing over all lamina of the volume, the total path length per second of muons traversing volume V is

$$\sum RAt\left\langle\frac{1}{\cos\theta}\right\rangle = RV\left\langle\frac{1}{\cos\theta}\right\rangle, \tag{3}$$

noting that $V = \sum At$.

Example: At the Earth's surface [1],

$$\frac{dR}{d\cos\theta} \approx \begin{cases} 0.06\cos^2\theta/\text{cm}^2/\text{s} & (0 < \theta < \pi/2), \\ 0 & (\pi/2 < \theta < \pi). \end{cases} \tag{4}$$

The total rate of muons incident on a horizontal surface is

$$R = \int_0^1 \frac{dR}{d\cos\theta}\, d\cos\theta \approx 0.02/\text{cm}^2/\text{s}. \tag{5}$$

The angular average is

$$\left\langle \frac{1}{\cos\theta} \right\rangle = \frac{1}{R} \int_0^1 \frac{1}{\cos\theta} \frac{dR}{d\cos\theta}\, d\cos\theta = \frac{0.06}{0.02} \int_0^1 \frac{\cos^2\theta}{\cos\theta}\, d\cos\theta = \frac{3}{2}\,, \tag{6}$$

so the total path length per second is

$$\frac{3}{2} RV = 0.03V, \tag{7}$$

for V in cm^3.

For an application of this result, see [2].

3 References

[1] T.K. Gaisser *et al.*, *Cosmic Rays*, http://pdg.lbl.gov/2006/reviews/cosmicrayrpp.pdf

[2] S. Palestini and K.T. McDonald, *Space Charge in Ionization Detectors* (March 25, 2007), http://puhep1.princeton.edu/~mcdonald/examples/spacecharge.pdf