Avoiding TPC Spatial Mapping Errors

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Coordinate Mapping in a TPC

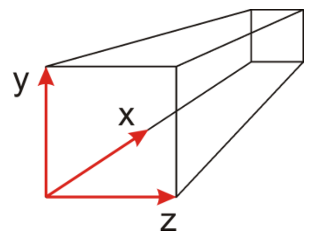
$(x_A, y_A, z_A) = f(x_I, y_I, z_I)$

ideal TPC has $f = Identity$

which is true if $\vec{E} = (0, 0, E_{z0})$ for all (x_I, y_I, z_I)

Consider small field deviations from this ideal

$$E_x = \delta E_x(x_I, y_I, z_I)$$
$$E_y = \delta E_y(x_I, y_I, z_I)$$
$$E_z = E_{z0} + \delta E_z(x_I, y_I, z_I)$$



The equation of motion for drift in a field is

$\vec{v} = \mu \vec{E}$

and to first order the solutions are

$$x_F(x_I, y_I, z) = x_I + \int_{Cathode}^{z} \frac{\delta E_x(x_I, y_I, z)}{E_{z0}} dz$$

$$y_F(x_I, y_I, z) = y_I + \int_{Cathode}^{z} \frac{\delta E_y(x_I, y_I, z)}{E_{z0}} dz$$

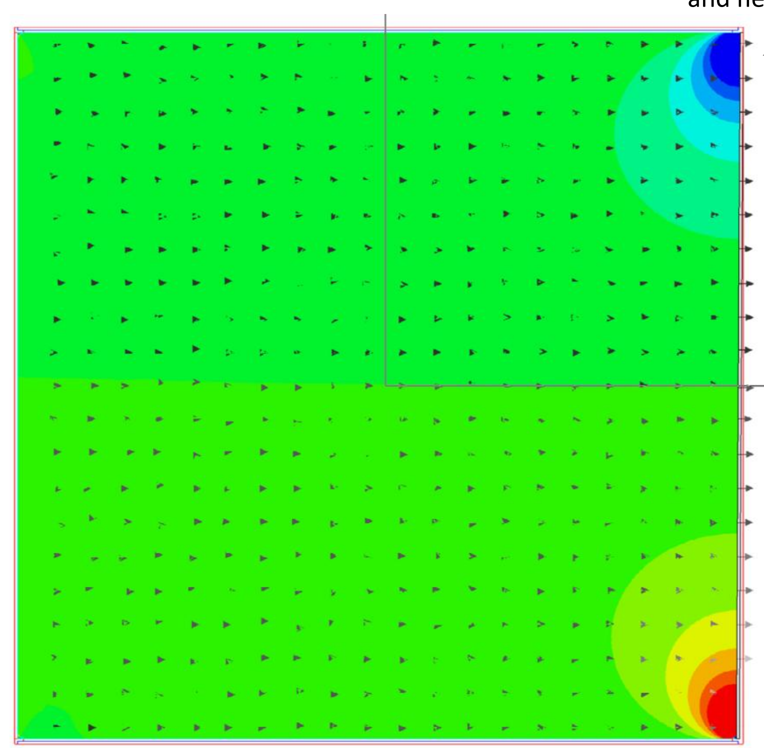
$$z_F(x_I, y_I, z) = z_I$$

and iterate ...

Use Maxwell 2D to compute the electric field, given the boundary conditions and space charge.
Output is (z, y, E_z, E_y).
Use curl E = 0 to obtain E_x when E_y varies linearly with x:

$\vec{E} = (\beta\, y, \beta\, x, E_0)$ has $\nabla \times \vec{E} = 0$

Field error caused by 1 cm gap between cathode and field cages



Taxonomy of Sources of TPC Mapping Distortions

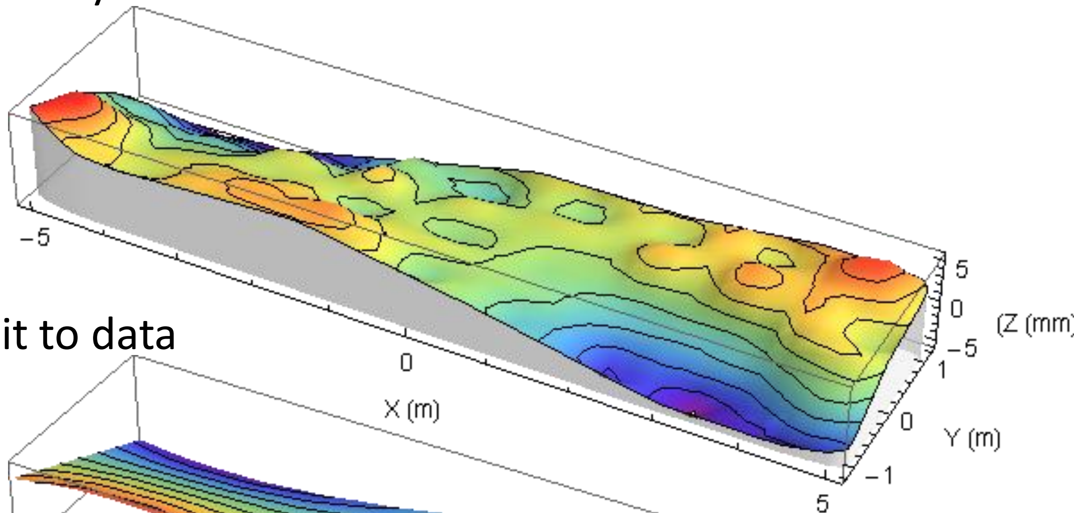
I. Geometric
  1. Cathode
    a. Displacement (Δz)
    b. Tilt [$z=\delta y$]
    c. Tip [$z=\gamma x$]
    d. Cup [$z=O(y)^2$]
    e. Bow [$z=O(x)^2$]
    f. Twist [$z=f(x,y)$]
    g. …
  2. Field Cage
    a. Skew
    b. Non-coplanarity
    c. …
  3. Anode
    a. Displacement
    b. Tilt
    c. Tip
    d. …

II. Electrostatic
  1. FC Gradient
    a. Differential (e.g. bad resistor)
    b. Integral (global error)
  2. Space Charge
    a. No liquid flow
    b. With liquid flow
  3. …

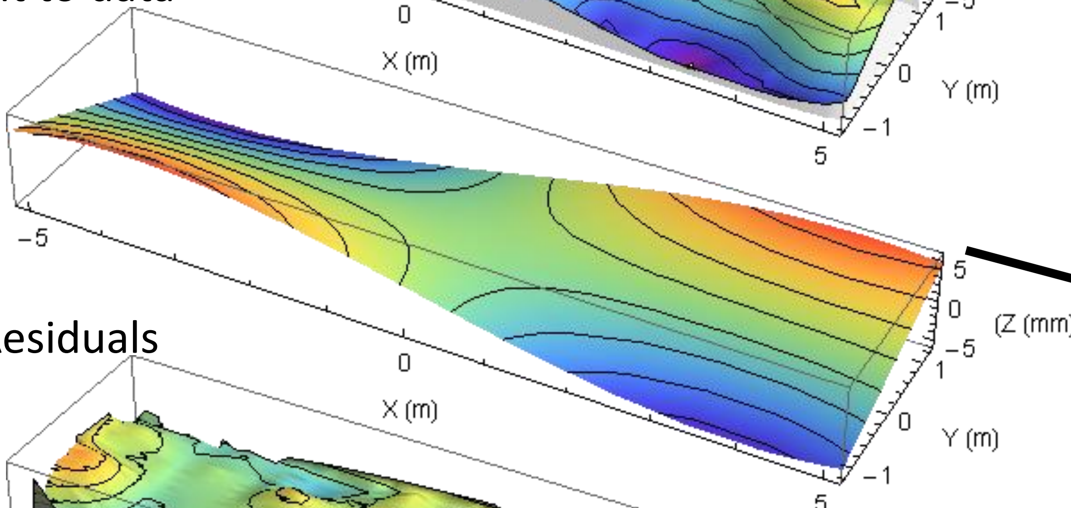
Bold items discussed here

Measured Cathode Sheet Distortion

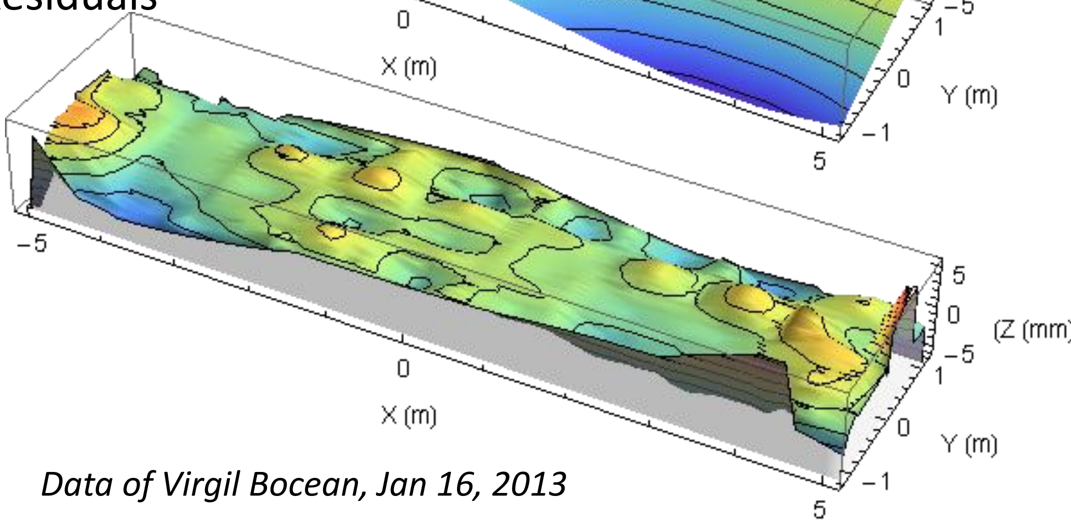
Survey data



Fit to data



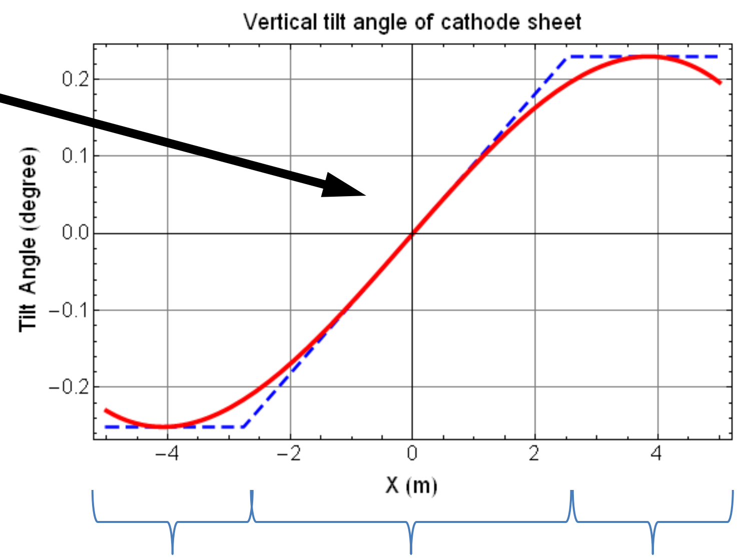
Residuals



Data of Virgil Bocean, Jan 16, 2013

This data is the deviations of the cathode sheet from a reference plane. The reference plane is unknown and undefined with respect to the field cage plane(s).

For these calculations we will **assume** that the reference plane is normal to the field cage plane.



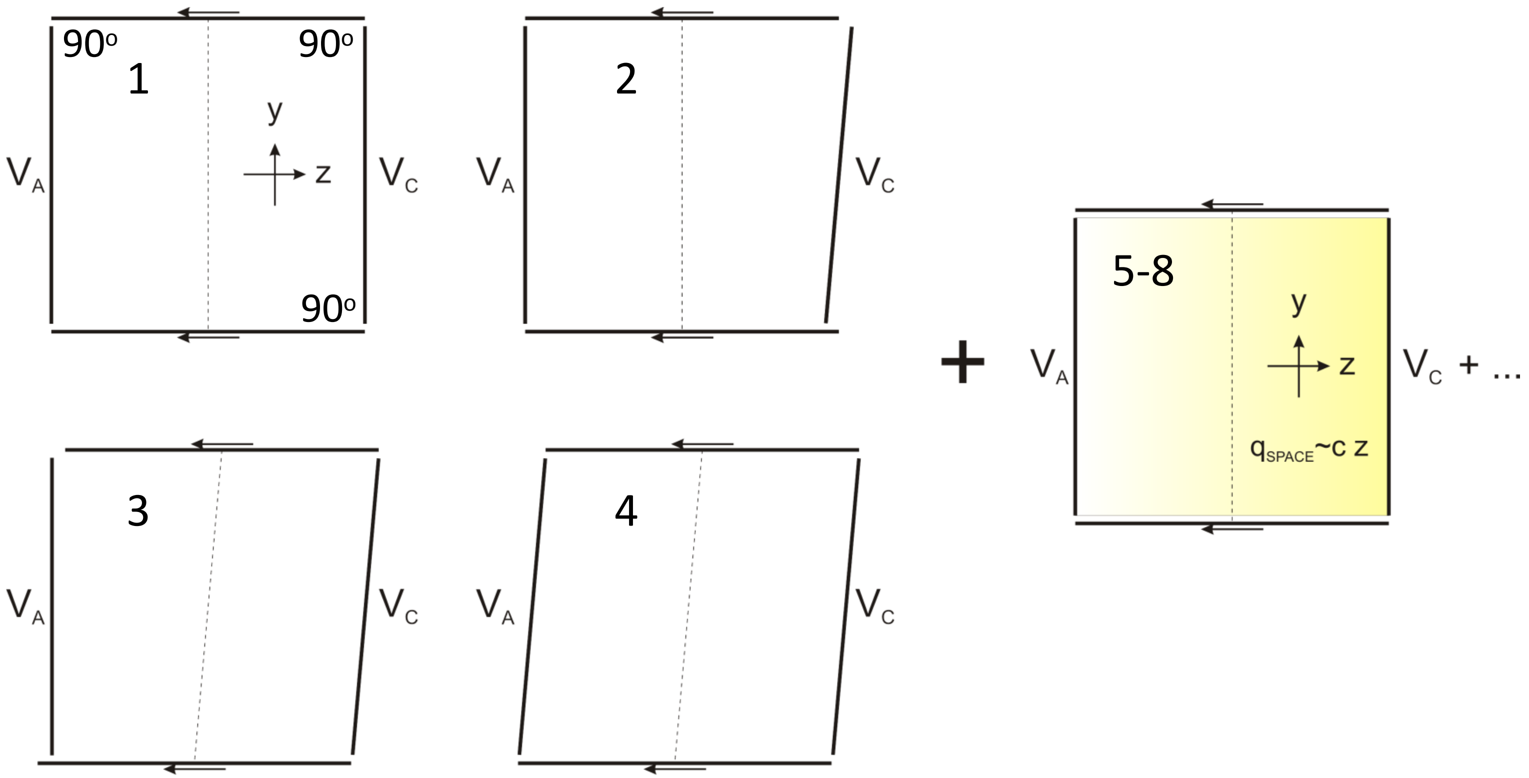
$\vec{E} = (0, 2.5\beta, E_0)$ $\vec{E} = (\beta\, y, \beta\, x, E_0)$ $\vec{E} = (0, 2.5\beta, E_0)$

with $\beta = 0.0018 E_0$

The TPC Geometric Distortions to be Modeled

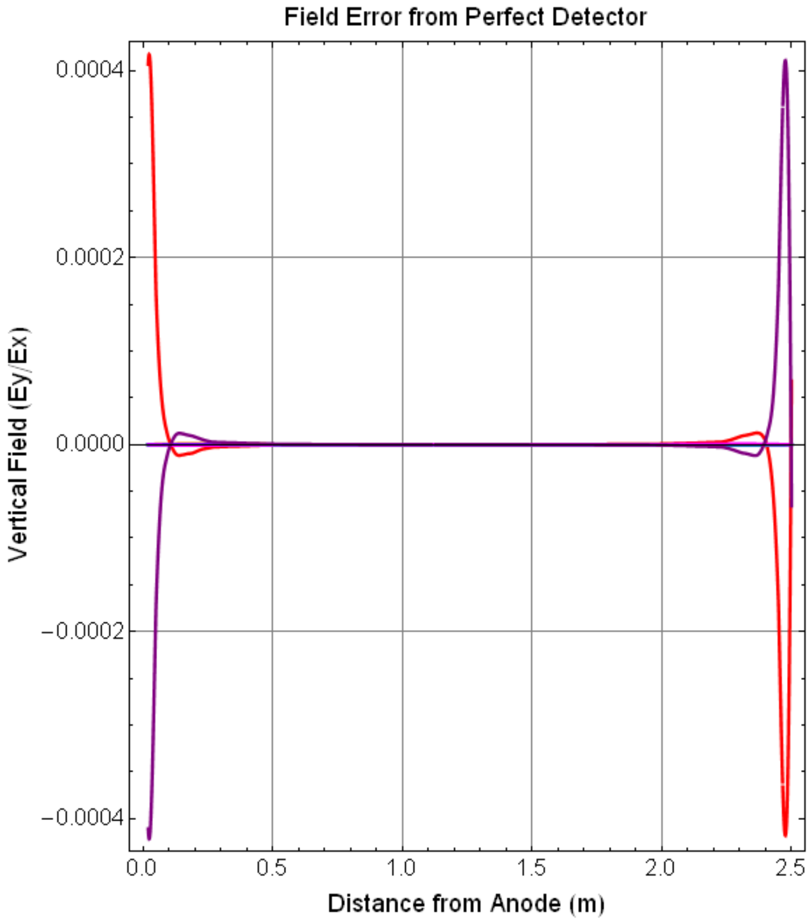
Compute E field for:

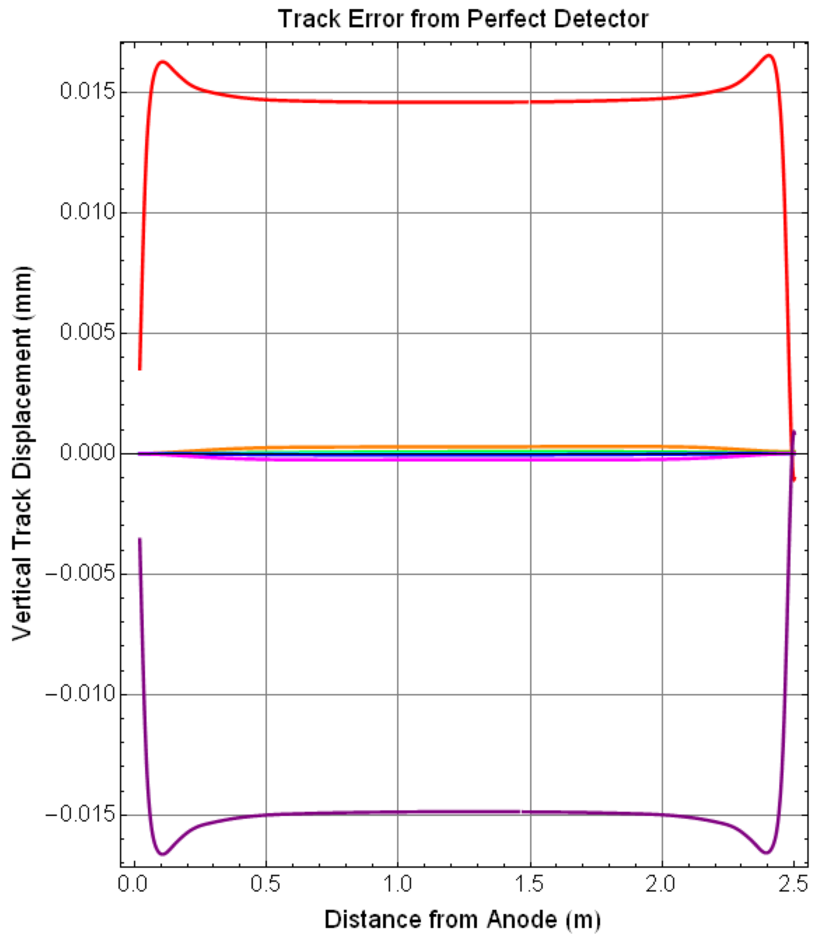
1 Perfect cathode, anode, and field cage
2 Rotated cathode
3 Rotated cathode + skewed field cage
4 Rotated cathode + rotated anode + skewed field cage
1'-4' Each of the above + space charge from cosmic rays (use superposition)



TPC Mapping for "Perfect" Geometry

Calculations for perfect rectilinear shape, proper alignment of field cage gradients
A aberration is caused by a gap between the cathode and anode sheets and the field cage, introduced to make it easier to rotate the anode and cathode. Otherwise the mapping is Identity.
Lines are drawn for y = -1.2, -0.8, -0.4, 0, 0.4, 0.8, and 1.2 m





Note: should be (Ey/Ez) on all graphs

Space Charge in LAr

Produced by Drifting Positive Ions from Cosmic Ray Tracks

See Palestini and McDonald, MicroBooNE docdb
Space Charge in Ionization Detectors
and Jostlein and McDonald, MicroBooNE docdb
Path Length of Muons Traversing an Arbitrary Volume

$$L_{TOTAL} = \frac{3}{2} R_H V = 300 V\ m^{-2} s^{-1}$$

$$\rho(x) = \frac{1.33\times10^{-7}}{\sqrt{1+\frac{23.0}{x^2}}}$$

$$\rho(x,y,z) = \frac{Kx}{\mu E(x)}$$

$$E[x] = E_0\sqrt{\left(\frac{E_A}{E_0}\right)^2 + \alpha^2\left(\frac{x}{D}\right)^2}$$

$$\alpha = \frac{D}{E_0}\sqrt{\frac{K}{\epsilon\mu}}$$

$$\epsilon = 1.505\,\epsilon = 1.328\times10^{-11} C\ m^{-1} V^{-1}$$

$$\mu = 1.6\times10^{-7}\ m^2 V^{-1} s^{-1}$$

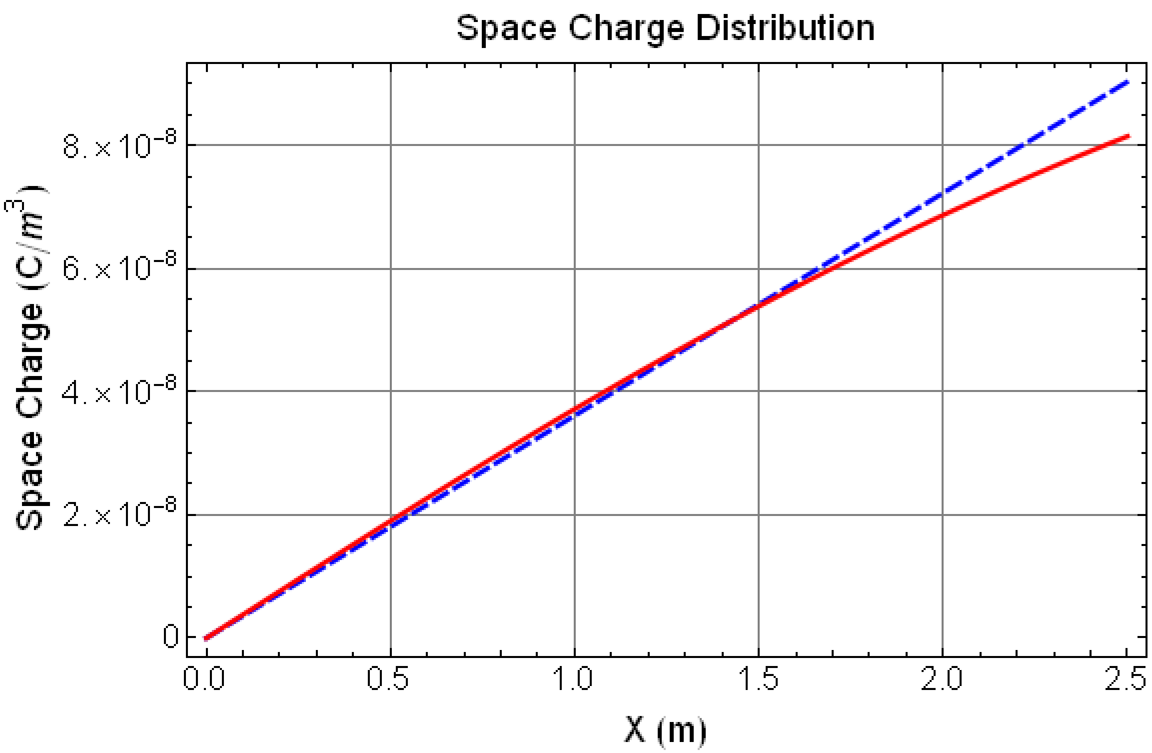
$$K = 2.89\times10^{-10}\ C\ m^{-1}$$

$$\alpha = 0.583385$$

$$D = 2.5\ m$$

$$E_0 = 50{,}000\ V\ m^{-1}$$

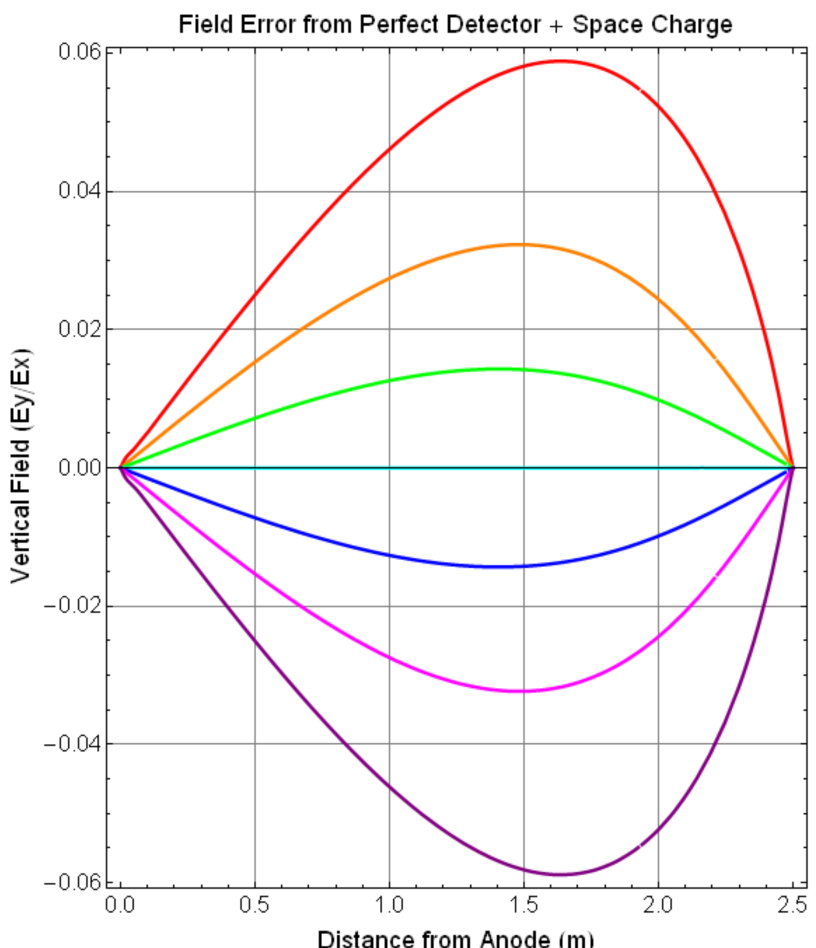
$$E_0 / E_A = 0.942896$$

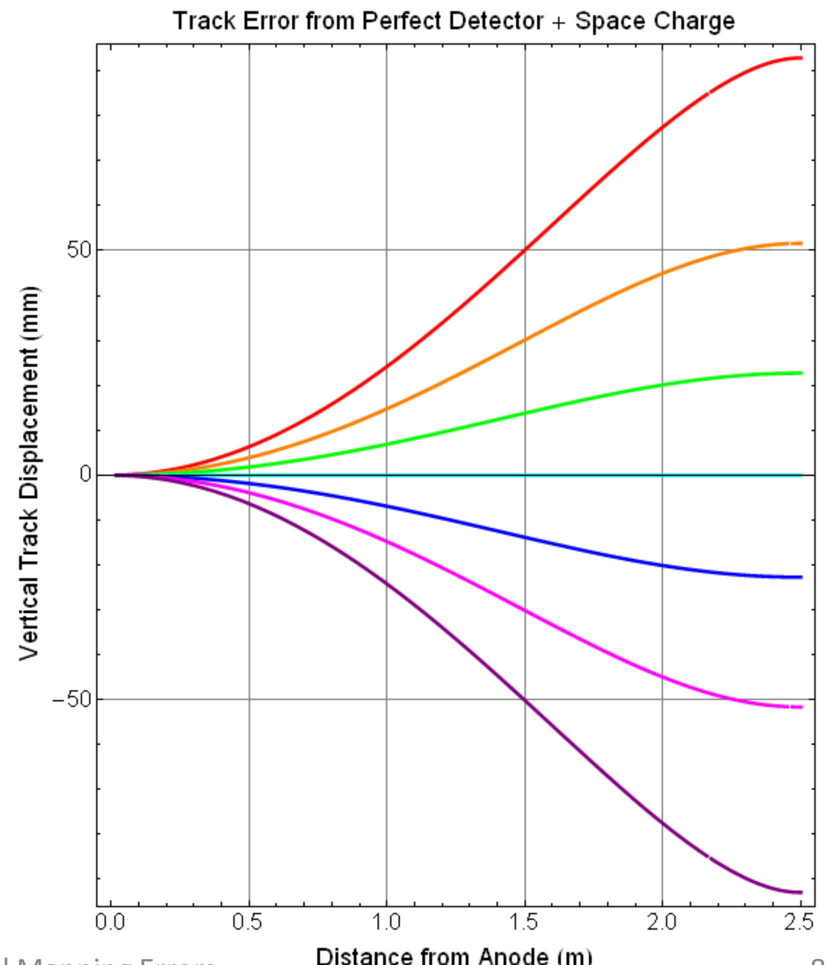


TPC Mapping for "Perfect" Geometry Plus Space Charge

Perfect rectilinear shape, proper alignment of field cage gradients, with space charge

Lines are drawn for y = -1.2, -0.8, -0.4, 0, 0.4, 0.8, and 1.2 m

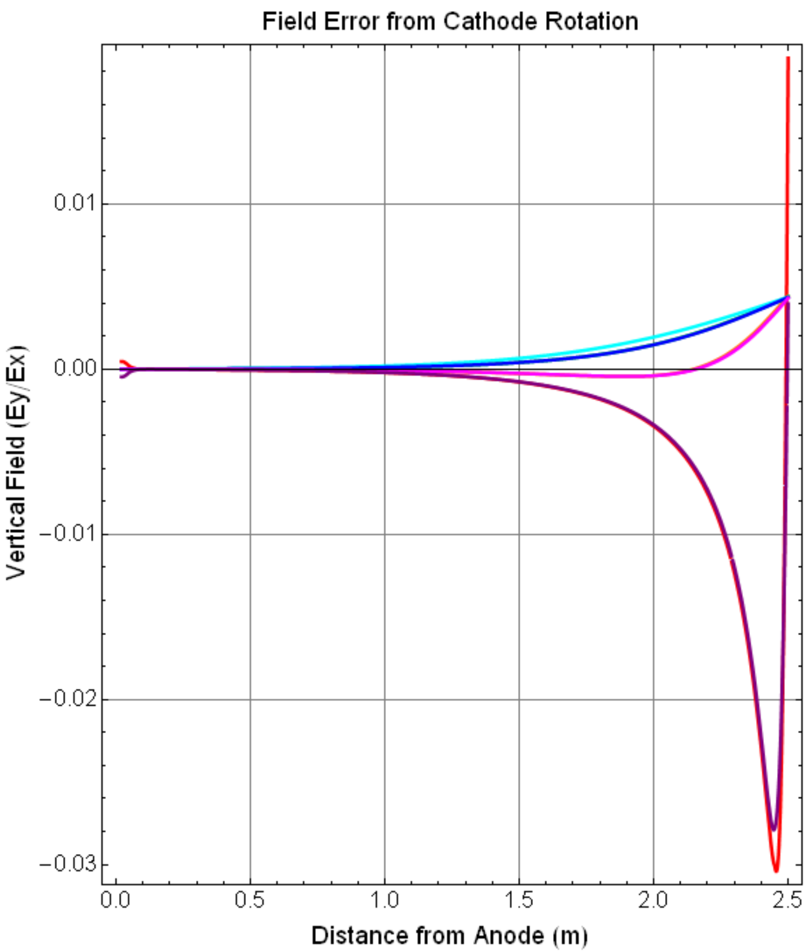


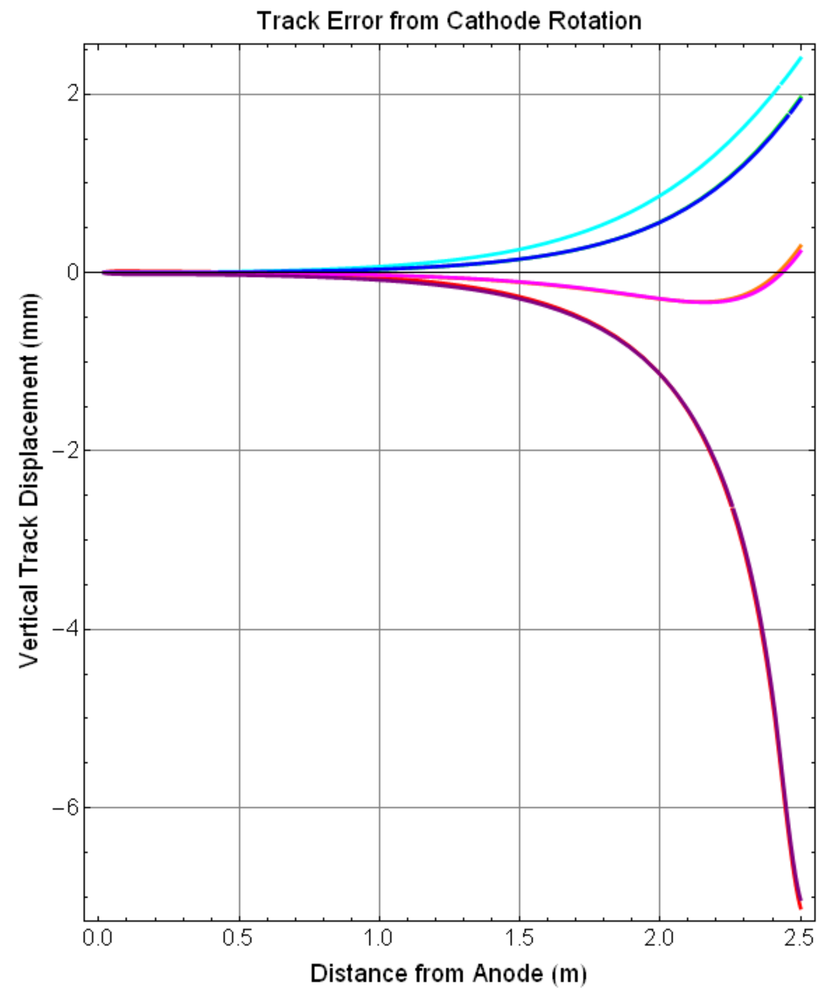


TPC Mapping for Cathode Rotation

Cathode is rotated by 0.25° maintaining proper alignment of field cage gradients

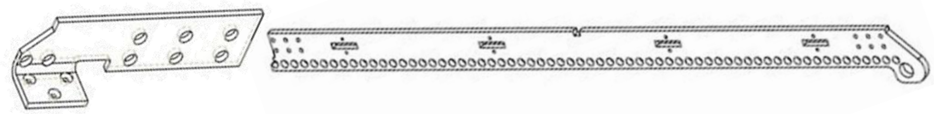
Lines are drawn for y = -1.2, -0.8, -0.4, 0, 0.4, 0.8, and 1.2 m



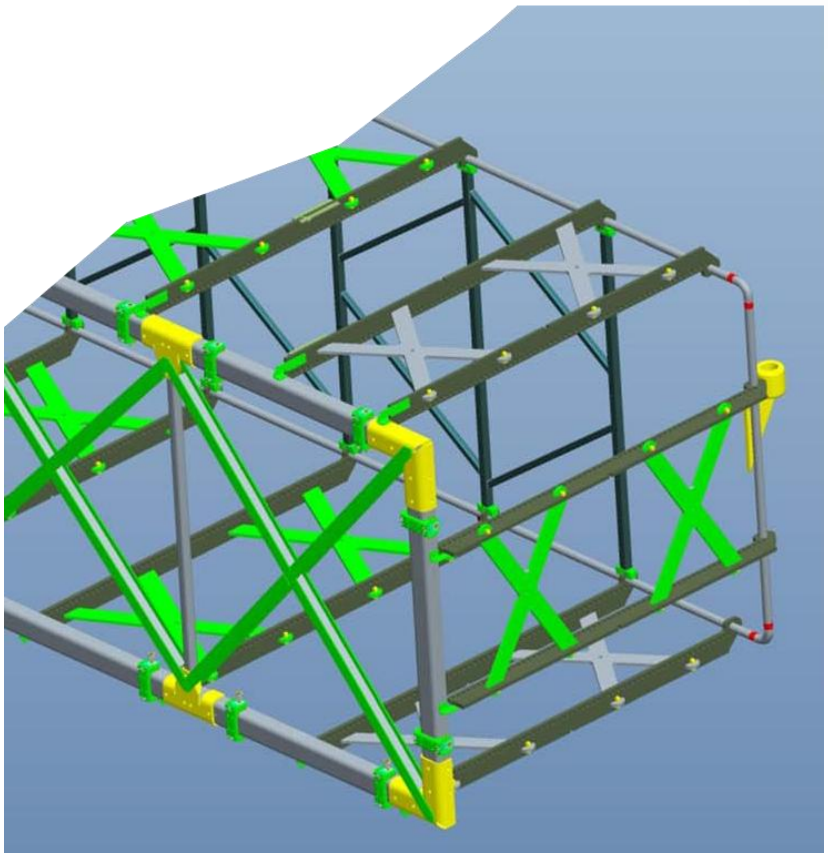


Constraints on TPC Deformations

Anode side



Cathode side

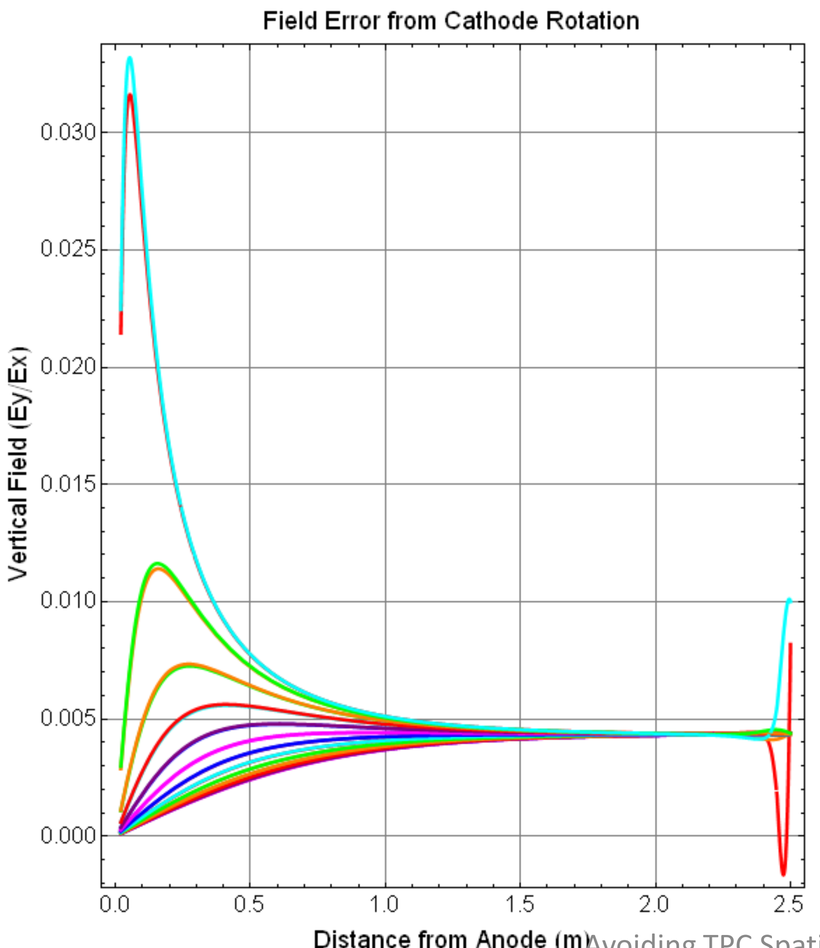


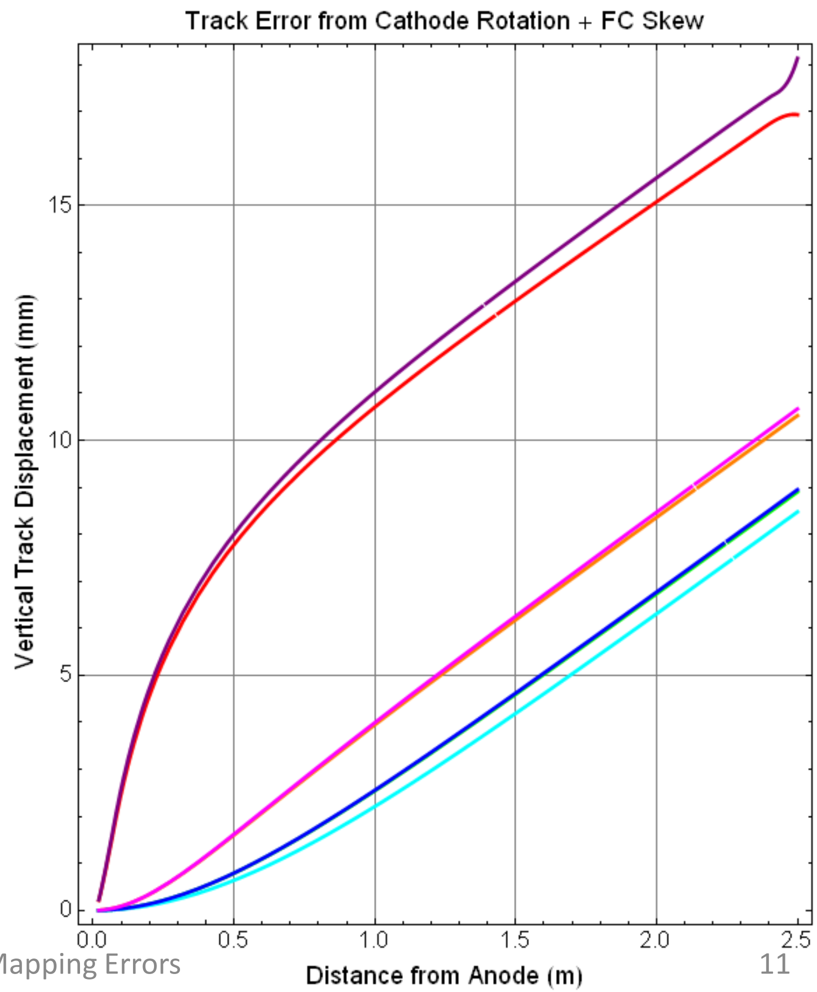
The tubing frame for the cathode sheet and the field cage tubes are constrained to move together as a unit by the G10 beams (dwg #354). This means that the previous solution, with only the cathode sheet tilted, is unlikely for a twisted cathode, and the next solution, with the top and bottom field cages displaced (skewed) is more reasonable.

The anode (wire) plane will be adjusted independently of the field cage and the cathode sheet, to ensure that it is flat (so that the wire plane spacing is constant everywhere.) It could still be tilted wrt the normal to the field cage, leading to a tracking error in the y direction.

TPC Mapping for Cathode Rotation + Field Cage Skew

Cathode is rotated by 0.25^o, top and bottom field cages are skewed right and left to follow
This is the most likely case for the observed twist in the cathode sheet

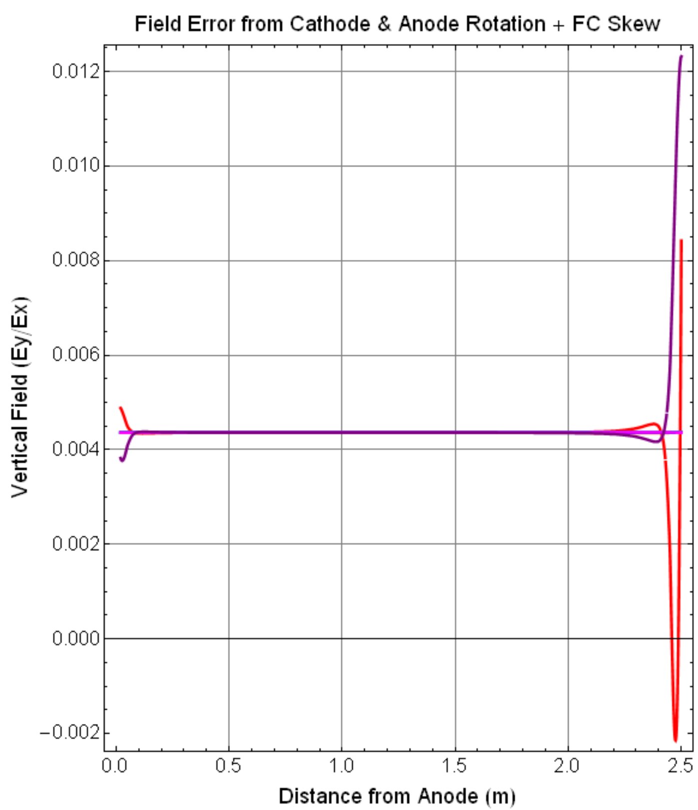


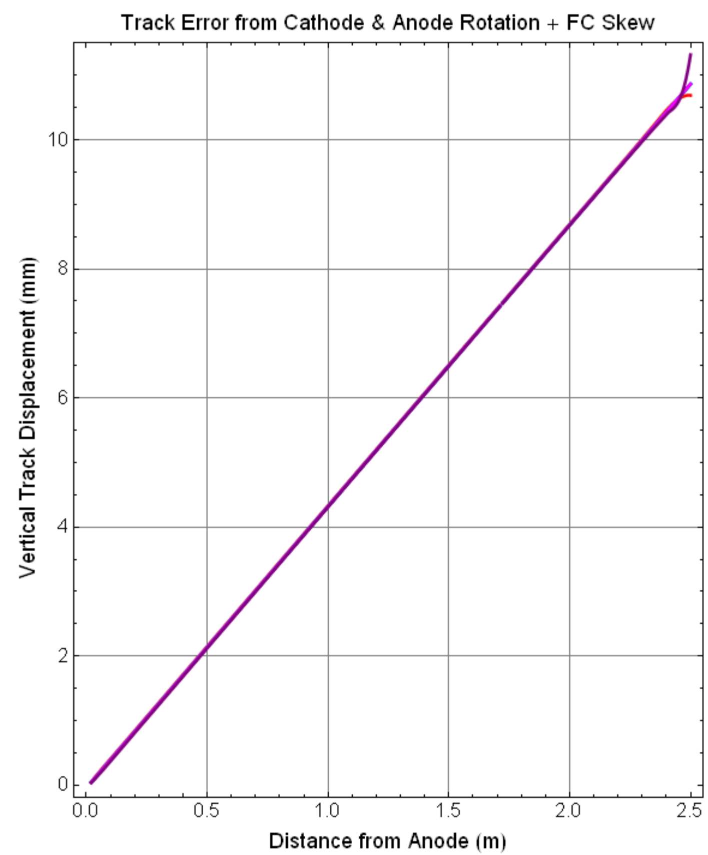


TPC Mapping for Cathode Rotation + Field Cage Skew + Anode Rotation

Cathode and anode are rotated by 0.25°, top and bottom field cages are skewed right and left to follow. This is the simple case: drift lines are straight lines are perpendicular to cathode and anode, and therefore are inclined at an angle of 0.25° to the horizontal. Y coordinate mapping is a shear function (y displacement proportional to z).

Since the anode plane will not be allowed to have a twist (so that the wire plane spacing is constant), this solution has an additional x dependence.

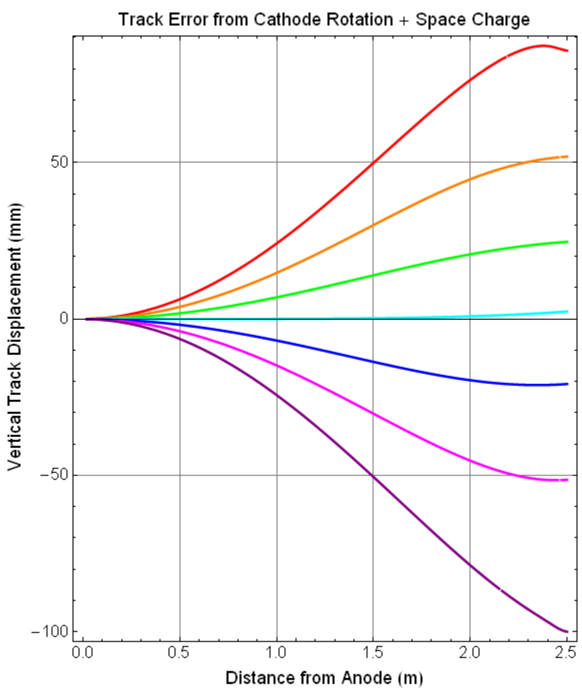


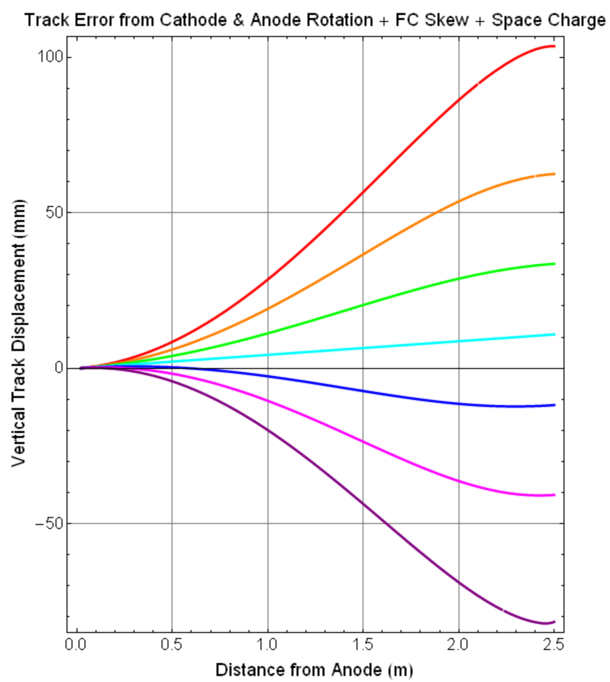


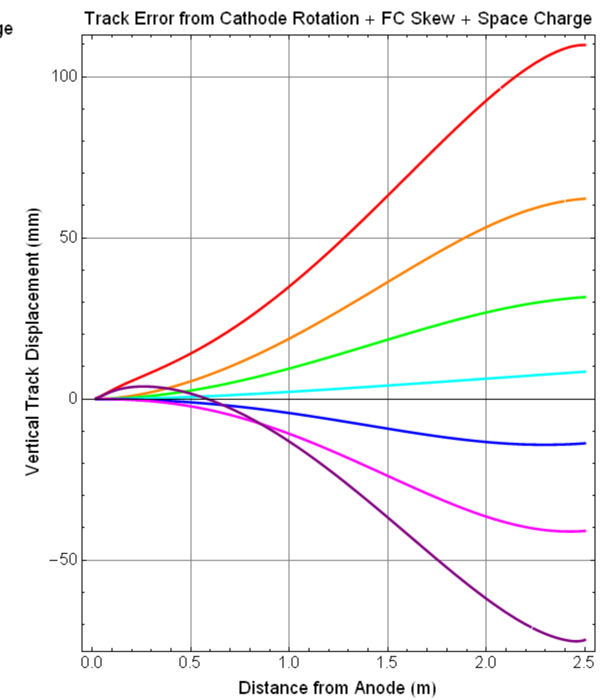
Combine Geometric Distortion and Space Charge

Superpose the solutions for geometric distortions and space charge.

The space charge distortion dominates for rotation angles below ArcTan(100/2500) = 2.3°

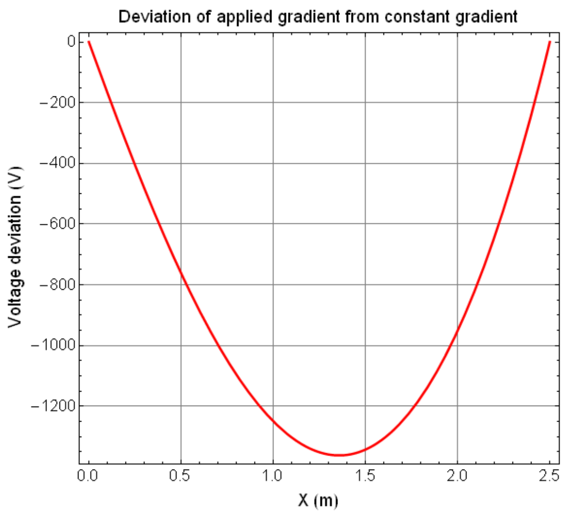


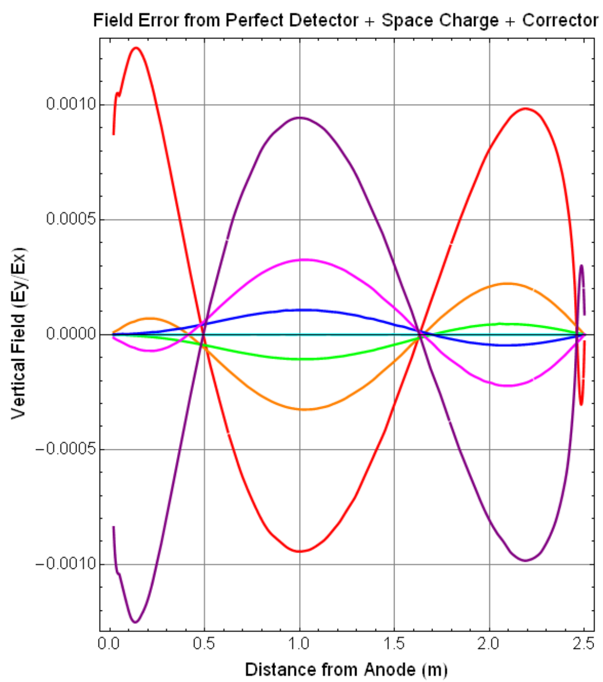


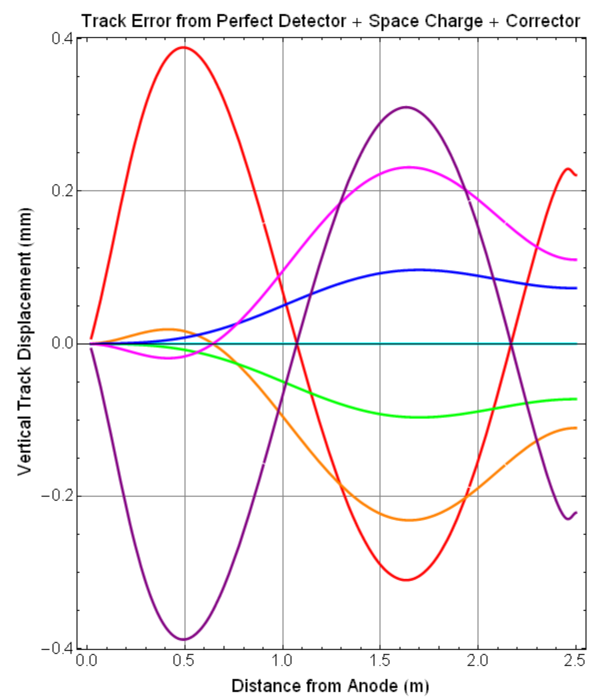


Adjusting the Field Cage Gradient:
Don't like space charge distortion? Eliminate it!

Perfect rectilinear shape + space charge with
varying field gradient: $V(x) = 48328\,x\,(1 + 0.0875\,(x + 1.25)^{3/2})$
Maximum track distortion is about 250x smaller
Of course, LAr flow complicates this.

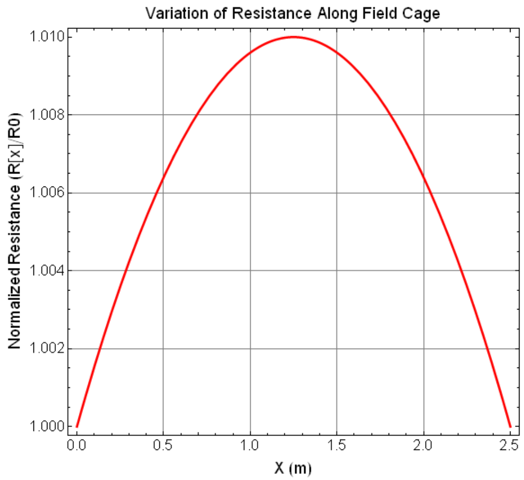


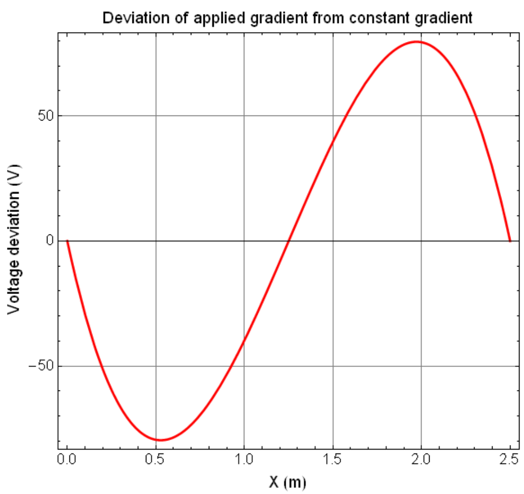


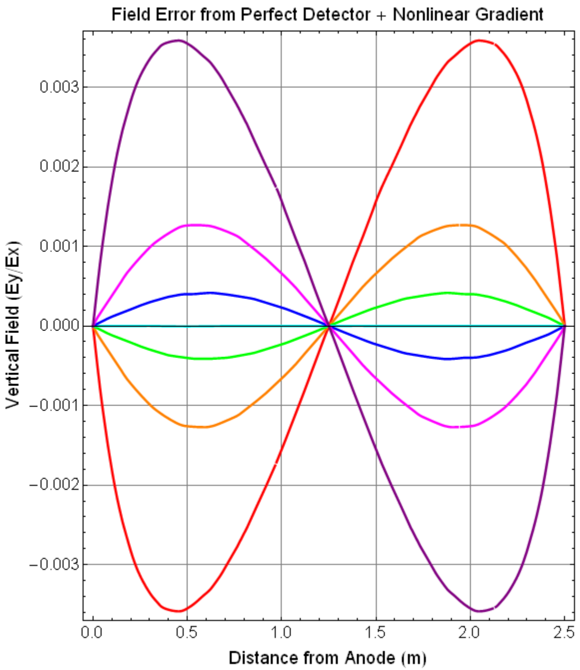


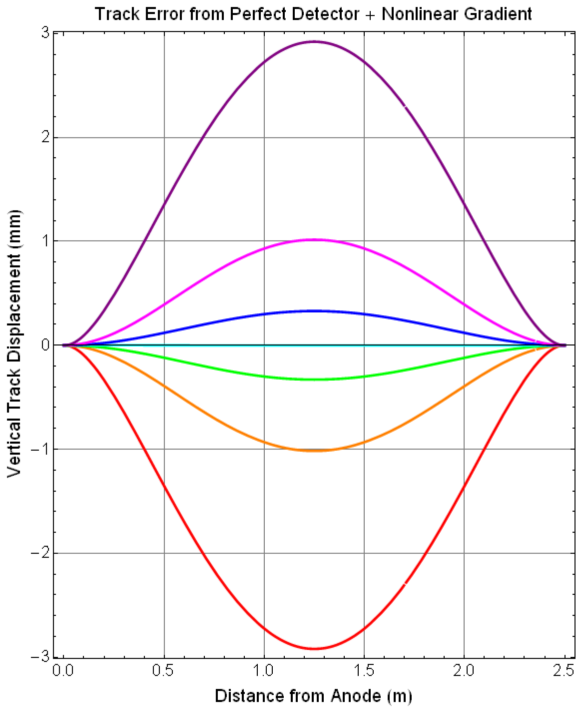
Integral Nonlinearity of Field Cage Voltage

Perfect rectilinear shape, but the resistors producing the voltage gradient on the filed cage vary quadratically with position. Maximum resistance variation is 1%.



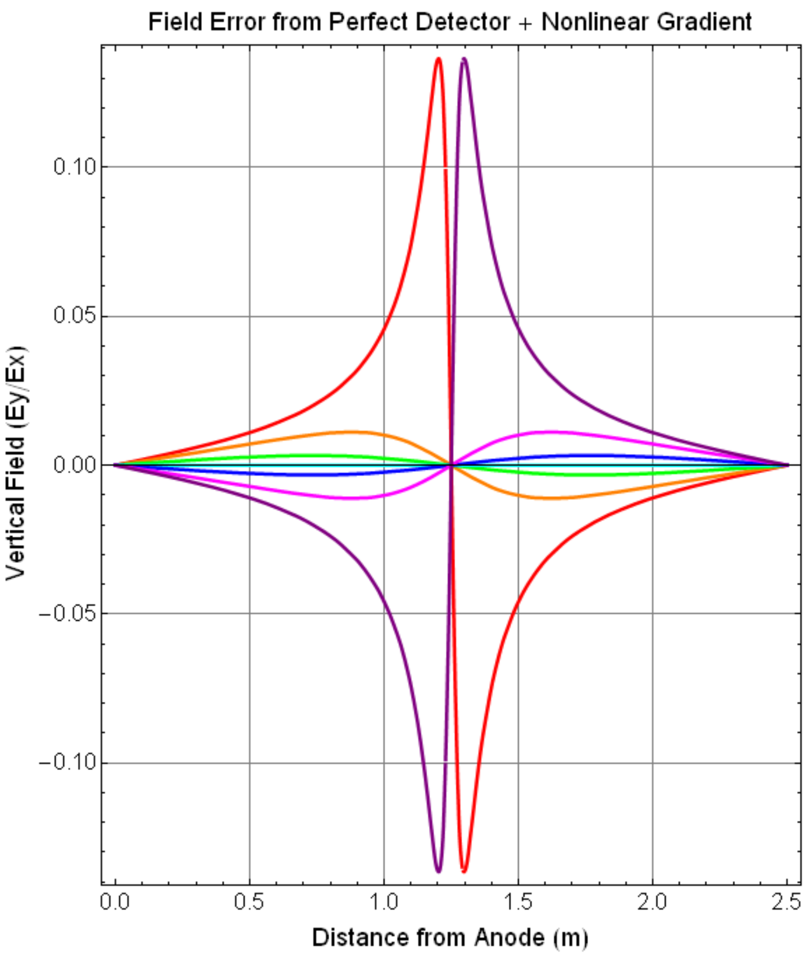


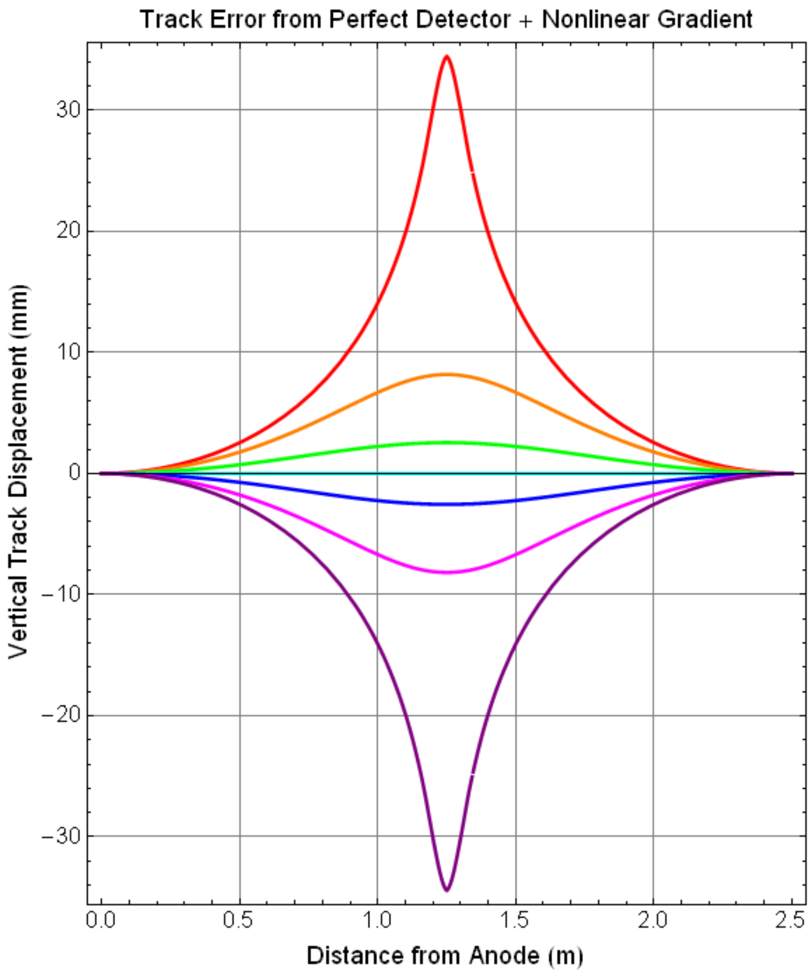




Integral Nonlinearity of Field Cage Voltage

Perfect rectilinear shape, but the central resistor is shorted in the resistor chain producing the voltage gradient on the filed cage.





Simple Determination of Field Cage Skew

Measuring the absolute cathode tilt angle and FC skew

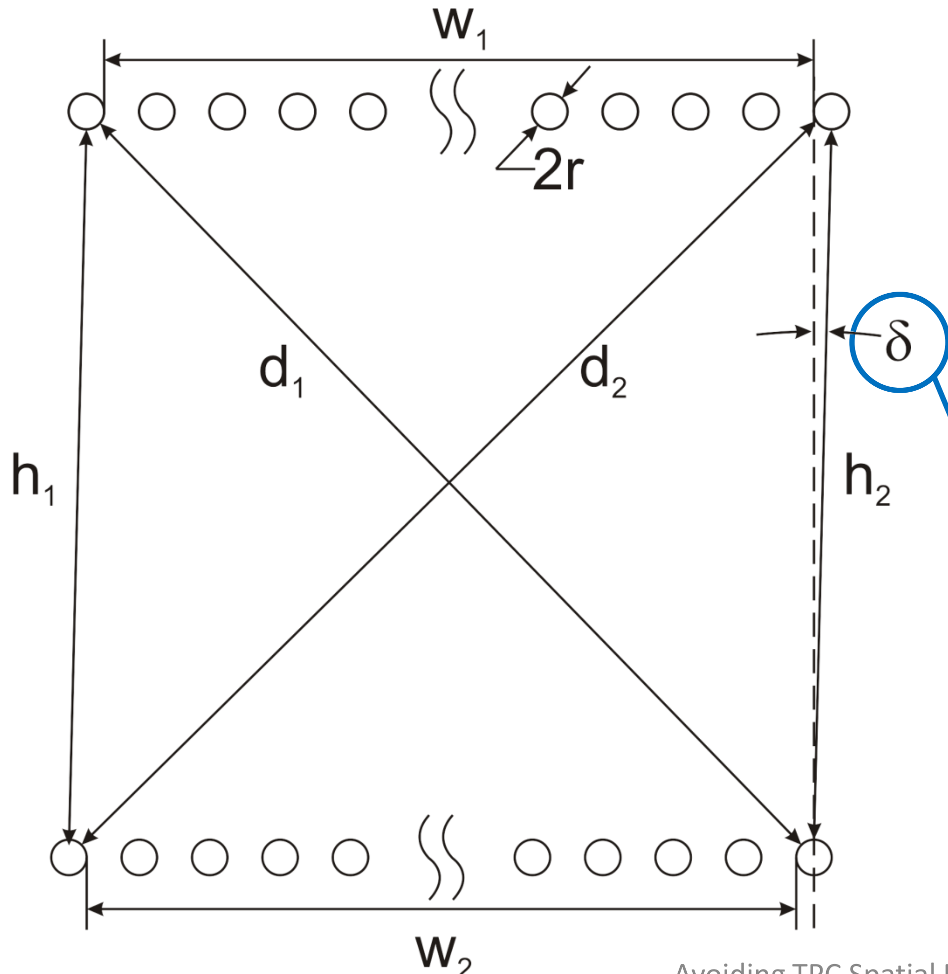
If h1 = h2 and w1 = w2 then deformation is simple skew, and we find tilt angle.
Otherwise the deformation includes trapezoidal terms.

$$\delta = ArcSin\left(\frac{D_2^{\,2} - D_1^{\,2}}{2\sqrt{2}H\sqrt{D_1^{\,2} + D_2^{\,2} - 2H^2}}\right)$$

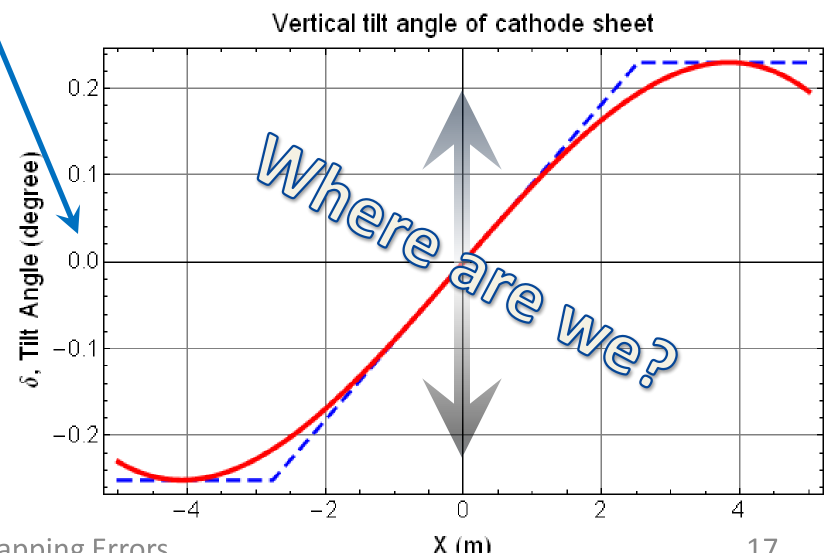
$$\approx \frac{\epsilon}{2H\sqrt{1-(H/D)^2}}$$

$$\Delta z_{Skew} = \pm\delta\, H/2$$

$$with\ D_i = d_i + 2r,\ H = h + 2r,\ and\ \epsilon = d_2 - d_1$$



Measure δ along the TPC (x)



Summary

- The impact of the mechanical tolerance of the TPC is probably negligible compared with the expected positive ion space charge, but we should invest some additional effort to ensure this. At least measurements of FC skew and coplanarity and cathode and anode tilt should be made.
- The laser calibration system should be able to correct the distortions in the TPC caused by geometrical and space charge distortions. Igor and Michele should comment.
- The flatness of the wire planes is an import parameter and should be maintained to xxx to ensure the wire plane spacing is within 0.5mm of the designed value.

Taken from docdb 2266: *MicroBooNE TPC Tolerance Issues*, Bo Yu, with words in red added